Chapter No 14
FREEZING

Q.1 Derive the Planks equation to compute freezing time of rectangular slab.

Plank has derived an approximate solution for the time of freezing which is often sufficient for engineering purpose.

Assumptions:

1. Initially all the food is at freezing temperature but is unfrozen.
2. The thermal conductivity of frozen part is constant.
3. All the material freezes at the freezing point with a constant latent heat.

Let a = thickness of the slab,

x = thickness of frozen layer,

T_a= temperature of the freezing medium,

T = freezing temperature of the foodstuff,

Ts= the surface temperature of the foodstuff,

K = thermal conductivity of frozen food,

λ = latent heat of frozen food,

ρ= density of unfrozen material,

t= time of freezing

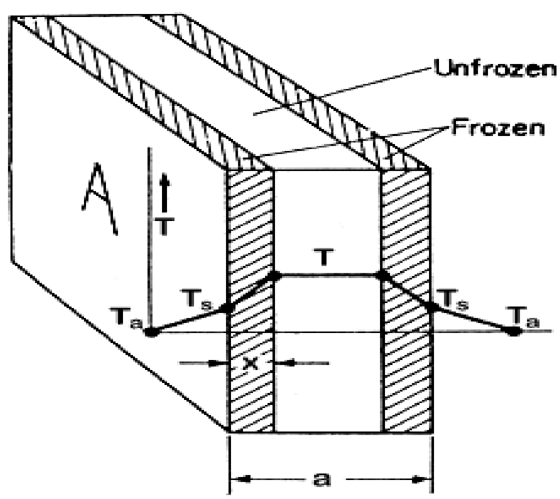

Fig. 8.1 Freezing of slab

The heat leaving by convection at time t is q W.

$$q = h_s A(T_s - T_a)$$

$$T_s = \frac{q}{h_s A} + T_a \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

Where, h_s = total surface heat transfer coefficient and A = heat transfer area.

$$\frac{1}{h_s} = \frac{1}{h_c} + \frac{x}{k} + \frac{1}{h_r}$$

Where, h_c=Convection heat transfer coefficient

h_r=Radiation heat transfer coefficient

Also the heat is conducted through the frozen layer of x thickness at steady state is,

$$q = \frac{KA}{x}(T_f - T_s)$$

$$T_s = T_f - \frac{qx}{KA} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

From equation 1 and 2

$$\frac{q}{h_s A} + T_a = T_f - \frac{qx}{KA}$$

$$\frac{q}{h_s A} + \frac{qx}{KA} = T_f - T_a$$

$$q\left(\frac{1}{h_s} + \frac{x}{K}\right) = A(T_f - T_a)$$

$$q = \frac{A(T_f - T_a)}{\left(\frac{1}{h_s} + \frac{x}{K}\right)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

In a given time dt sec, a layer thickness dx of material freezes then,

$$q = \frac{A\,dx\,\rho\,\lambda}{dt}$$

$$q = A\,\rho\,\lambda \frac{dx}{dt} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4)$$

Equating equation 3 and 4

$$\frac{A(T_f - T_a)}{\left(\frac{1}{h_s} + \frac{x}{K}\right)} = A\,\rho\,\lambda \frac{dx}{dt}$$

$$\frac{(T_f - T_a)}{\left(\frac{1}{h_s} + \frac{x}{K}\right)} = \rho\lambda\frac{dx}{dt}$$

$$dt = \rho\lambda\frac{1}{(T_f - T_a)}\left(\frac{1}{h_s} + \frac{x}{K}\right)dx$$

Now if the thickness of the slab is a, the time taken for the center of the slab at x=a/2 to freeze can be obtained by integrating x=0 to x=a/2 during which time t goes from t to t_f

$$\int_0^{t_f} dt = \frac{\rho\lambda}{(T_f - T_a)}\int_0^{a/2}\left(\frac{1}{h_s} + \frac{x}{K}\right)dx$$

$$(t)_0^{t_f} = \frac{\rho\lambda}{(T_f - T_a)}\left[\left(\frac{x}{h_s} + \frac{x^2}{2K}\right)\right]_0^{a/2}$$

$$(t_f - 0) = \frac{\rho\lambda}{(T_f - T_a)}\left(\frac{a/2}{h_s} + \frac{\left[\frac{a}{2}\right]^2}{2K}\right)$$

$$t_f = \frac{\rho\lambda}{(T_f - T_a)}\left(\frac{a}{2h_s} + \frac{a^2}{8K}\right)$$

This is the required Plank equation for the freezing time. The General equation can be written as follows for different shapes.

$$t_f = \frac{\rho\lambda}{(T_f - T_a)}\left(\frac{Pa}{h_s} + \frac{Ra^2}{K}\right)$$

Shape	**P**	**R**
Slab	1/2	1/8
Long cylinder	1/4	1/16
Sphere	1/6	1/24

Problem 1: If a slab of meat is to be frozen between refrigerated plates with the plate temperature at -34°C, how long will it take to freeze if the slab is 10 cm thick and the meat is wrapped in cardboard 1 mm thick on either side of the slab? What would be the freezing time if the cardboard were not present?

Assume that for the plate freezer, the surface heat-transfer coefficient is 600 J m^{-2} s^{-1} °C^{-1}, the thermal conductivity of cardboard is 0.06 J m^{-1} s^{-1}° C^{-1} the thermal conductivity of frozen meat is 1.6 J m^{-1} s^{-1} °C^{-1}, its latent heat is 2.56 x 10^5 J kg^{-1} and density 1090 kg m^{-3}. Assume also that meat freezes at -2°C.

Given

1. T_a=-34°C
2. a = 10 cm = 0.1 m
3. x = 1 mm = 0.001 m
4. h_c=600 J m^{-2} s^{-1} °C^{-1}
5. Kc = 0.06 J m^{-1} s^{-1}° C^{-1}
6. K =1.6 J m^{-1} s^{-1} °C^{-1}
7. λ =2.56 x 10^5 J kg^{-1}
8. ρ =1090 kg m^{-3}
9. T_f =-2°C
10. t_f = ?

Sol:

A. With Cardboard

1. The total surface heat transfer coefficient (hs)

$$\frac{1}{h_s} = \frac{1}{h_c} + \frac{x}{kc} + \frac{1}{h_r}$$

$$\frac{1}{h_s} = \frac{1}{600} + \frac{0.001}{0.06} + 0$$

$$\frac{1}{h_s} = 0.019$$

$$h_s = 52.6 \text{ J m}^{-2}\text{ s}^{-1}\text{ °C}^{-1}$$

2. Freezing time (t_f)

$$t_f = \frac{\rho\lambda}{(T_f - T_a)}\left(\frac{Pa}{h_s} + \frac{Ra^2}{K}\right)$$

For plate freezer, P=1/2 =0.5, R=1/8=0.125

$$t_f = \frac{1090 \text{ x } 2.56 \text{ x} 10^5}{(-2-(-34))}\left(\frac{0.5 \text{ x } 0.1}{52.6} + \frac{0.125 \text{ x } 0.1^2}{1.6}\right)$$

$$t_f = 8720000(9.5057 \text{ x } 10^{-4} + 7.8125 \text{ x } 10^{-4})$$

$$t_f = 15101.20 \text{ s}$$

$$t_f = 4.2 \text{ h}$$

B. Without Cardboard

1. The total surface heat transfer coefficient (hs)

$$\frac{1}{h_s} = \frac{1}{h_c} + \frac{x}{kc} + \frac{1}{h_r}$$

$$\frac{1}{h_s} = \frac{1}{600} + 0 + 0$$

$$\frac{1}{\text{h}_\text{s}} = 1.7 \text{ x } 10^{-3}$$

h_s= 588.2352 J m^{-2} s^{-1} $°C^{-1}$

2. Freezing time (t_f)

$$t_f = \frac{\rho\,\lambda}{(T_f - T_a)}\left(\frac{Pa}{h_s} + \frac{Ra^2}{K}\right)$$

For plate freezer, P=1/2 =0.5, R=1/8=0.125

$$t_f = \frac{1090 \text{ x } 2.56 \text{ x}10^5}{(-2-(-34))}\left(\frac{0.5 \text{ x } 0.1}{588.2352} + \frac{0.125 \text{ x } 0.1^2}{1.6}\right)$$

$$t_f = 8720000(8.5000 \text{ x } 10^{-5} + 7.8125 \text{ x } 10^{-4})$$

$$t_f = 7553.700 \text{ s}$$

$$t_f = 2.1 \text{ h}$$

Ans:

1. **The time of freezing with Cardboard = 4.2 h**
2. **The time of freezing without Cardboard = 2.1 h**

Problem 2: Slabs of meat 0.0635 m thick are to be frozen in an air blast freezer at 244.3 K (-28.9 ^{0}C). The meat is initially at the freezing temperature of 270.4 K (-2.8^{0}C). The meat contains 75% moisture. The heat transfer coefficient is h=17 W/m^2K. The physical properties are ρ = 1057 kg/m^3 for the unfrozen meat and k = 1.038 W/m K for the unfrozen meat. Calculate the freezing time. Latent heat of fusion of water to ice is 335 kJ/kg.

Given

1. T_a=244.3 K
2. a =0.0635 m
3. h=17 W/m^2K
4. ρ = 1057 kg/m^3
5. λ = 335 kJ/kg
6. Moisture content=75%
7. K = 1.038 W/m K
8. t_f = ?

Sol:

Since the latent heat of fusion of water to ice is 335 kJ/kg, for meat with 75% moisture,

$$\lambda = 0.75 \text{ x } 335$$

$$\lambda = 251.2 \text{ kJ/kg}$$

Freezing time (t_f)

$$t_f = \frac{\rho\,\lambda}{(T_f - T_a)}\left(\frac{Pa}{h_s} + \frac{Ra^2}{K}\right)$$

For plate freezer, P=1/2 =0.5, R=1/8=0.125

$$t_f = \frac{1057 \text{ x } 2.512 \text{ x}10^5}{(270.4 - 244.3)}\left(\frac{0.5 \text{ x } 0.0635}{17} + \frac{0.125 \text{ x } 0.0635^2}{1.038}\right)$$

$$t_f = 2.395 \text{ x } 10^4$$

$$t_f = 6.65 \text{ h}$$

Ans: The Freezing time of the meat = 6.65 h.