how efficiently the centrifuge separates the solid from the liquid phase. This principle of solids purity being dependent on the degree of separation of the solid from the liquid phase applies not only in crystallization but also in solvent extraction.

In order to solve this problem, the saturation concentration of sugar in water at 20°C, and the water content of the crystals fraction after centrifugation must be known.

3.1.3 System Boundaries

Figure 3.2 shows how the boundaries of the system can be moved to facilitate solving the problem. If the boundary completely encloses the whole process, there will be one stream entering and four streams leaving the system. The boundary can also be set just around the evaporator in which case there is one stream entering and two leaving. The boundary can also be set around the centrifuge or around the drier. A material balance can be carried out around any of these subsystems or around the whole system. The material balance equation may be a total mass balance or a component balance.

3.1.4 Total Mass Balance

The equation in section "Law of Conservation of Mass," when used on the total weight of each stream entering or leaving a system, represents a total mass balance. The following examples illustrate how total mass balance equations are formulated for systems and subsystems.

Example 3.1. In an evaporator, dilute material enters and concentrated material leaves the system. Water is evaporated during the process. If I is the weight of the dilute material entering the system, W is the weight of water vaporized, and C is the weight of the concentrate, write an equation that represents the total mass balance for the system. Assume that a steady state exists.

Solution:

The problem statement describes a system depicted in Fig. 3.3.
The total mass balance is

Inflow = Outflow + Accumulation

$$I = W + C \text{ (accumulation is 0 in a steady-state system)}$$

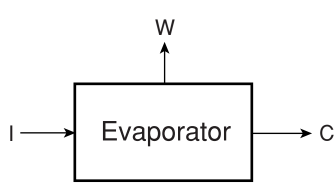

Figure 3.3 Input and exit streams in an evaporation process.

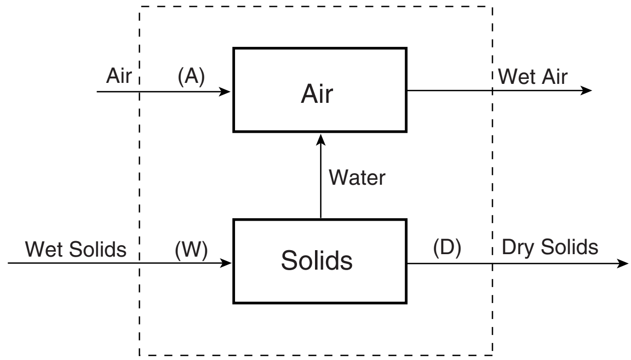

Figure 3.4 Diagram of material flow in a dehydration process.

Example 3.2. Construct a diagram and set up a total mass balance for a dehydrator. Air enters at the rate of A lb/min, and wet material enters at W lb/min. Dry material leaves the system at D lb/min. Assume steady state.

Solution:

The problem statement describes a system (dehydrator) where air and wet material enters and dry material leaves. Obviously, air must leave the system also, and water must leave the system. A characteristic of a dehydrator not written into the problem statement is that water removed from the solids is transferred to air and leaves the system with the air stream. Figure 3.4 shows the dehydrator system and its boundaries. Also shown are two separate subsystems—one for the solids and the other for air—with their corresponding boundaries. Considering the whole dehydrator system, the total mass balance is

$$\text{W} + \text{A} = \text{wet air} + \text{D}$$

Considering the air subsystem:

$$\text{A} + \text{water} = \text{wet air}$$

The mass balance for the solids subsystem is

$$\text{W} = \text{water} + \text{D}$$

Example 3.3. Orange juice concentrate is made by concentrating single-strength juice to 65% solids followed by dilution of the concentrate to 45% solids using single-strength juice. Draw a diagram for the system and set up mass balances for the whole system and for as many subsystems as possible.

Solution:

The problem statement describes a process depicted in Fig. 3.5. Consider a hypothetical proportionator that separates the original juice (S) to that which is fed to the evaporator (F) and that (A) which is used to dilute the 65% concentrate. Also, introduce a blender to indicate that part of the process where

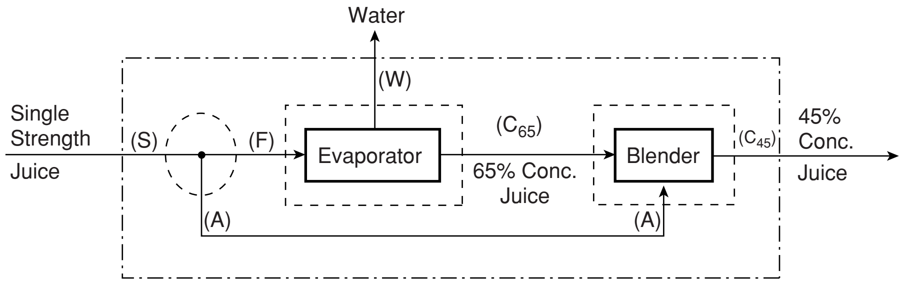

Figure 3.5 Diagram of an orange juice concentrate process involving evaporation and blending of concentrate with freshly squeezed juice.

the 65% concentrate (C_{65}) and the single-strength juice are mixed to produce the 45% concentrate (C_{45}). The material balance equations for the whole system and the various subsystems are:

Overall: $S = W + C_{45}$
Proportionator: $S = F + A$
Evaporator: $F = W + C_{65}$
Blender: $C_{65} + A = C_{45}$

3.1.5 Component Mass Balance

The same principles apply as in the total mass balance except that components are considered individually. If there are n components, n independent equations can be formulated; one equation for total mass balance and $n - 1$ component balance equations.

Because the object of a material balance problem is to identify the weights and composition of various streams entering and leaving a system, it is often necessary to establish several equations and simultaneously solve these equations to evaluate the unknowns. It is helpful to include the known quantities of process streams and concentrations of components in the process diagram in order that all streams where a component may be present can be easily accounted for. In a material balance, use mass units and concentration in mass fraction or mass percentage. If the quantities are expressed in volume units, convert to mass units using density.

A form of a component balance equation that is particularly useful in problems involving concentration or dilution is the expression for the mass fraction or weight percentage.

$$\text{Mass fraction A} = \frac{\text{mass of component A}}{\text{total mass of mixture containing A}}$$

Rearrange the equation:

$$\text{Total mass of mixture containing A} = \frac{\text{mass of component A}}{\text{mass fraction of A}}$$

Thus, if the weight of component A in a mixture is known, and its mass fraction in that mixture is known, the mass of the mixture can be easily calculated.

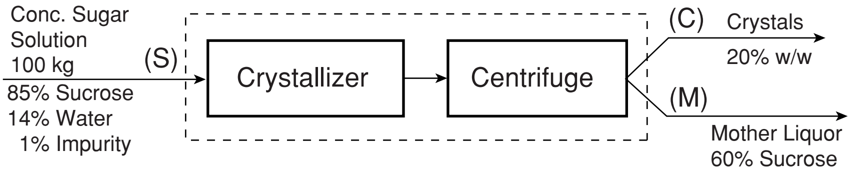

Figure 3.6 Diagram showing composition and material flow in a crystallization process.

Example 3.4. Draw a diagram and set up a total mass and component balance equation for a crystallizer where 100 kg of a concentrated sugar solution containing 85% sucrose and 1% inert, water-soluble impurities (balance, water) enters. Upon cooling, the sugar crystallizes from solution. A centrifuge then separates the crystals from a liquid fraction, called the mother liquor. The crystal slurry fraction has, for 20% of its weight, a liquid having the same composition as the mother liquor. The mother liquor contains 60% sucrose by weight.

Solution:

The diagram for the process is shown in Fig. 3.6. Based on a system boundary enclosing the whole process of crystallization and centrifugation, the material balance equations are as follows.

Total mass balance:

$$S = C + M$$

Component balance on sucrose:

$$S(0.85) = M(0.6) + C(0.2)(0.6) + C(0.8)$$

The term on the left is sucrose in the inlet stream. The first term on the right is sucrose n the mother liquor. The second term on the right is sucrose in mother liquor carried by crystals. The last term on the right is sucrose in the crystals.

Component balance on water:

Let x = mass fraction of impurity in the mother liquor

$$S(0.14) = M(0.4 - x) + C(0.2)(0.4 - x)$$

The first term on the left is water in the inlet stream. The first term on the right is water in the mother liquor. The last term on the right is water in the mother liquor adhering to the crystals.

Component balance on impurity:

$$S(0.01) = M(x) + C(0.2)(x)$$

Note that a total of four equations can be formulated but there are only three unknown quantities (C, M, and x). One of the equations is redundant.

Example 3.5. Draw a diagram and set up equations representing total mass balance and component mass balance for a system involving the mixing of pork (15% protein, 20% fat, and 63% water) and backfat (15% water, 80% fat, and 3% protein) to make 100 kg of a mixture containing 25% fat.

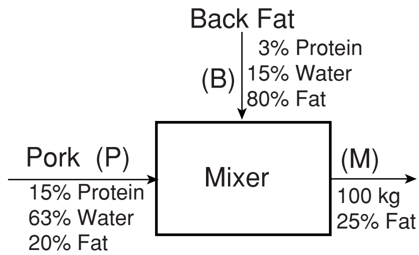

Figure 3.7 Composition and material flow in a blending process.

Solution:

The diagram is shown in Fig. 3.7.
Total mass balance:

$$P + B = 100$$

Fat balance:

$$0.2\ P + 0.8\ B = 0.25(100)$$

These two equations are solved simultaneously by substituting $P = 100 - B$ into the second equation.

$$0.2(100 - B) + 0.8\ B = 25$$

$$B = \frac{25 - 20}{0.8 - 0.2} = 8.33\ \text{kg}$$

$$P = 100 - 8.33 = 91.67\ \text{kg}$$

3.1.6 Basis and “Tie Material”

A “tie material” is a component used to relate the quantity of one process stream to another. It is usually the component that does not change during a process. Examples of tie material are solids in dehydration or evaporation processes and nitrogen in combustion processes. Although it is not essential that these tie materials are identified, the calculations are often simplified if it is identified and included in one of the component balance equations. This is illustrated in Example 3.6 of the section “A Steady State” where the problem is solved rather readily using a component mass balance in the solid (the tie material in this system) compared to when the mass balance was made on water. In a number of cases, the tie material need not be identified as illustrated in examples in the section “Blending of Food Ingredients.”

A “basis” is useful in problems where no initial quantities are given and the answer required is a ratio or a percentage. It is also useful in continuous flow systems. Material balance in a continuous flow system is done by assuming as a basis a fixed time of operation. A material balance problem can be solved on any assumed basis. After all the quantities of process streams are identified, the specific quantity asked in the problem can be solved using ratio and proportion. It is possible to change basis when considering each subsystem within a defined boundary inside the total system.

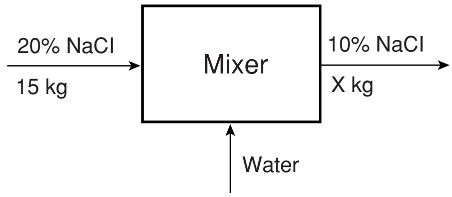

Figure 3.8 Composition and material flow for a dilution process.

3.2 MATERIAL BALANCE PROBLEMS INVOLVED IN DILUTION, CONCENTRATION, AND DEHYDRATION

3.2.1 Steady State

These problems can be solved by formulating total mass and component balance equations and solving the equations simultaneously.

Example 3.6. How many kilograms of a solution containing 10% NaCl can be obtained by diluting 15 kg of a 20% solution with water?

Solution:

The process diagram in Fig. 3.8 shows that all NaCl enters the mixer with the 20% NaCl solution and leaves in the diluted solution. Let x = kg 10% NaCl solution; y = kg water. The material balance equations are

Total mass: $15 = X - Y$
Component: $15(0.20) = X(0.10)$

The total mass balance equation is redundant because the component balance equation alone can be used to solve the problem.

$$x = \frac{3}{0.1} = 30 \text{ kg}$$

The mass fraction equation can also be used in this problem. Fifteen kilograms of a 20% NaCl solution contains 3 kg NaCl. Dilution would not change the quantity of NaCl so that 3 kg of NaCl is in the diluted mixture. The diluted mixture contains 10% NaCl, therefore:

$$x = \frac{3 \text{ kg NaCl}}{\text{mass fraction NaCl}} = \frac{3}{0.1} = 30 \text{ kg}$$

Example 3.7. How much weight reduction would result when a material is dried from 80% moisture to 50% moisture?

Solution:

The process diagram is shown in Fig. 3.9. Dehydration involves removal of water and the mass of solids remain constant. There are two components, solids and water, and a decrease in the concentration

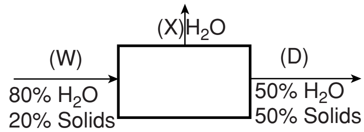

Figure 3.9 Composition and material flow in a dehydration process.

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of water, indicating a loss, will increase the solids concentration. Let: W = mass of 80% moisture material, D = mass of 50% moisture material, and X = mass water lost, which is also the reduction in mass.

No weights are specified, therefore express reduction in weight as a ratio of final to initial weight. The material balance equations are

Total mass: $W = X + D$
Water: $0.8W = 0.5\,D + X$

Solving simultaneously:

$$W = X + D; X = W - D$$
$$0.8\,W = 0.5\,D + (W - D)$$
$$0.5\,D = 0.2\,W;$$
$$\frac{D}{W} = \frac{0.2}{0.5} = 0.4$$
$$\%\text{wt. reduction} = \frac{W - D}{W}(100) = \left(1 - \frac{D}{W}\right)100$$
$$= (1 - 0.4)(100) = 60\%$$

The problem can also be solved by using as a basis 100 kg of 80% moisture material. Let W = 100.

Total mass balance: $100 = X + D$
Water balance: $0.8(100) = X + 0.5\,D$

Solving simultaneously, by subtracting one equation from the other:

$$100 - 80 = D(1 - 0.5);\ D = 20/0.5 = 40\text{ kg.}$$
$$\%\text{ wt. reduction} = \frac{100 - 40}{100}(100) = 60\%$$

3.2.2 Volume Changes on Mixing

When two liquids are mixed, the volumes are not always additive. This is true with most solutions and miscible liquids. Sodium chloride solution, sugar solutions, and ethanol solution all exhibit volume changes on mixing. Because of volume changes, material balances must be done on mass rather than volume of components. Concentrations on a volume basis must be converted to a mass basis before the material balance equations are formulated.

Example 3.8. Alcohol content in beverages are reported as percent by volume. A "proof" is twice the volume percent of alcohol. The density of absolute ethanol is 0.7893 g/cm^3. The density of a solution containing 60% by weight of ethanol is 0.8911 g/cm^3. Calculate the volume of absolute ethanol that must be diluted with water to produce 1 liter of 60% by weight, ethanol solution. Calculate the "proof" of a 60% ethanol solution.

Solution:

Use as a basis: 1 liter of 60% w/w ethanol.
Let X = volume of absolute ethanol in liters. The component balance equation on ethanol is:

$$X(1000)(0.7983) = 1(1000)(0.8911)(0.6)$$
$$X = 0.677 \text{ L}$$
$$\text{g water} = 1000(0.8911) - 0.677(0.7983) = 356.7 \text{ g or } 356.7 \text{ mL}$$

Total volume components = 0.677 + 0.3567 = 1.033 L

This is a dilution problem, and a component balance on ethanol was adequate to solve the problem. Note that a mass balance was made using the densities given. There is a volume loss on mixing as more than 1 liter of absolute ethanol and water produced 1 liter of 60% w/w ethanol solution.

To calculate the "proof" of 60% w/w ethanol, use as a basis 100 g of solution.

Volume of solution = 100/0.8911 = 112.22 cm^3
Volume of ethanol = 100(0.6)/0.7893 = 76.016 cm^3
Volume percent = (76.016/112.22)(100) = 67.74%
Proof = 2 (volume percent) = 135.5 proof

3.2.3 Continuous Versus Batch

Material balance calculations are the same regardless whether a batch or continuous process is being evaluated. In a batch system, the total mass considered includes what entered or left the system at one time. In a continuous system, a basis of a unit time of operation may be used and the material balance will be made on what entered or left the system during that period of time. The previous examples were batch operations. If the process is continuous, the quantities given will all be mass/time (e.g., kg/h). If the basis used is 1 hour of operation, the problem is reduced to the same form as a batch process.

Example 3.9. An evaporator has a rated evaporation capacity of 500 kg water/h. Calculate the rate of production of juice concentrate containing 45% total solids from raw juice containing 12% solids.

Solution:

The diagram of the process is shown in Fig. 3.10. Use as a basis 1 hour of operation. Five hundred kg of water leaves the system. A component balance on solids and a total mass balance will be needed to solve the problem. Let F = the feed, 12% solids juice, and C = concentrate containing 45% solids. The material balance equations are:

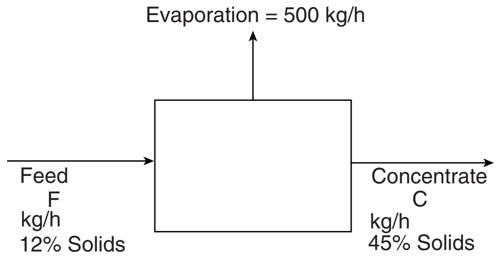

Figure 3.10 Flow diagram of an evaporation process.

Total mass:

$$F = C + 500$$

Solids:

$$0.12\,F = 0.45\,C;\; F = \frac{0.45\,C}{0.12} = 3.75\,C$$

$$C = \frac{500}{3.75 - 1} = 181.8\text{ kg}$$

Substituting and solving for C: $3.75\,C = C + 500$.

Because the basis is 1 hour of operation, the answer will be: Rate of production of concentrate = 181.8 kg/h

3.2.4 Recycle

Recycle is evaluated similarly as in the previous examples, but the boundaries of subsystems analyzed are moved around to isolate the process streams being evaluated. A system is defined that has a boundary surrounding the recycle stream. If this total system is analyzed, the problem may be reduced to a simple material balance problem without recycle.

Example 3.10. A pilot plant model of a falling film evaporator has an evaporation capacity of 10 kg water/h. The system consists of a heater through which the fluid flows down in a thin film and the heated fluid discharges into a collecting vessel maintained under a vacuum where flash evaporation reduces the temperature of the heated fluid to the boiling point. In continuous operation, a recirculating pump draws part of the concentrate from the reservoir, mixes this concentrate with feed, and pumps the mixture through the heater. The recirculating pump moves 20 kg of fluid/h. The fluid in the collecting vessel should be at the desired concentration for withdrawal from the evaporator at any time. If feed enters at 5.5% solids and a 25% concentrate is desired, calculate: (a) the feed rate and concentrate production rate, (b) the amount of concentrate recycled, and (c) concentration of mixture of feed and recycled concentrate.

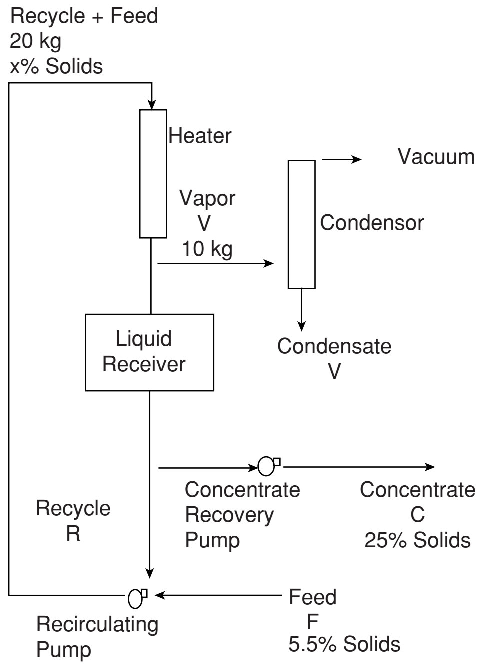

Figure 3.11 Diagram of material flow in a falling film evaporator with product recycle.

Solution:

A diagram of the system is shown in Fig. 3.11. The basis is 1 hour of operation.

A mass and solids balance over the whole system will establish the quantity of feed and concentrate produced per hour.

Total mass:

$$F = C + V;\ F = C + 10$$

Solids:

$$F(0.055) = C(0.25);\ F = C\left(\frac{0.25}{0.055}\right) = 4.545\,C$$

Substituting F:

$$4.545\,C = C + 10;\ C = \frac{10}{4.545 - 1} = 2.82\ \text{kg}$$

(a) Solving for F : F = 4.545(2.82) = 12.82 kg/h. The concentrate production rate is 2.82 kg/h.

(b) Material balance around the recirculating pump:

$$R + F = 20;\ R = 20 - 12.82 = 7.18\ \text{kg}$$

$$\text{Recycle rate} = 7.18\ \text{kg/h}$$

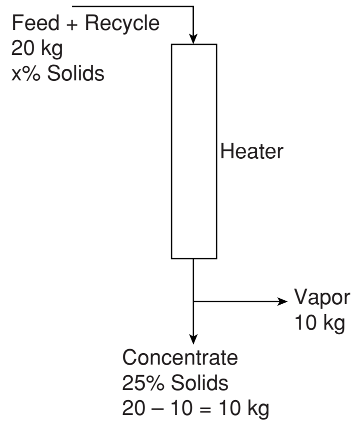

Figure 3.12 Diagram of material balance around the heater of a falling film evaporator.

(c) A material balance can be made around the part of the system where the vapor separates from the heated fluid as shown in Fig. 3.12.

Solids balance: $20(x) = 10(0.25);\ x = 2.5/20 = 0.125.$

The fluid entering the heater contains 12.5% solids.

3.2.5 Unsteady State

Unsteady-state material balance equations involve an accumulation term in the equation. Accumulation is expressed as a differential term of the rate of change of a variable with respect to time. A mass balance is made in the same manner as steady-state problems. Because a differential term is involved, the differential equation must be integrated to obtain an equation for the value of the dependent variable as a function of time.

Example 3.11. A stirred tank with a volume of 10 liters contains a salt solution at a concentration of 100 g/L. If salt-free water is continuously fed into this tank at the rate of 12 L/h, and the volume is maintained constant by continuously overflowing the excess fluid, what will be the concentration of salt after 90 min.

Solution:

This is similar to a dilution problem, except that the continuous overflow removes salt from the tank, thus reducing not only the concentration but also the quantity of salt present. A mass balance will be made on the mass of salt in the tank and in the streams entering and leaving the system.

Let $x =$ the concentration of salt in the vessel at any time. Representing time by the symbol t, the accumulation term will be dx/dt, which will have units of g salt/(L $\cong$ min). Convert all time units to minutes in order to be consistent with the units. Multiplying the differential term by the volume will result in units of g salt/min. The feed contains no salt, therefore the input term is zero. The overflow is at the same rate as the feed, and the concentration in the overflow is the same as inside the vessel, therefore the output term is the feed rate multiplied by the concentration inside the vessel. The mass balance equation is

$$0 = Fx + V\left(\frac{dx}{dt}\right); \frac{dx}{dt} = -\frac{F}{V}x$$

The negative sign indicates that x will be decreasing with time. Separating variables:

$$\frac{dx}{x} = -\frac{F}{V}dt$$

Integrating:

$$\ln x = -\left(\frac{F}{V}\right)t + C$$

The constant of integration is obtained by substituting $x = 100$ at $t = 0$; $C = \ln(100)$; $F = 12$ L/h = 0.2 L/min; $V = 10$ L

$$\ln\left(\frac{x}{100}\right) = -\frac{0.2\,t}{10}$$

At 90 min: $\ln(0.01\,x) = -1.800$.

$$x = 100(e^{-1.80}) = 16.53 \text{ g/L}$$

Example 3.12. The generation time is the time required for cell mass to double. The generation time of yeast in a culture broth has been determined from turbidimetric measurements to be 1.5 hours. (a) If this yeast is used in a continuous fermentor, which is a well-stirred vessel having a volume of 1.5 L, and the inoculum is 10,000 cells/mL, at what rate can cell-free substrate be fed into this fermentor in order that the yeast cell concentration will remain constant? The fermentor volume is maintained constant by continuous overflow. (b) If the feed rate is 80% of what is needed for a steady-state cell mass, calculate the cell mass after 10 hours of operation.

Solution:

This problem is a combination of dilution with continuous cell removal, but with the added factor of cell generation by growth inside the vessel. The diagram shown in Fig. 3.13 represents the material balance on cell mass around a fermentor. The balance on cell mass or cell numbers is as follows:

Input(from feed + cell growth) = output + accumulation

(a) The substrate entering the fermentor is cell free, therefore this term in the material balance equation is zero. Let R = substrate feed rate = overflow rate, because fluid volume in fermentor is maintained constant. Let x = cells/mL. The material balance on cell numbers with the appropriate time derivatives for rate of increase in cell numbers, $(dx/dt)_{gen}$, and accumulation $(dx/dt)_{acc}$ is

$$V\left[\frac{dx}{dt}\right]_{gen} = Rx + V\left[\frac{dx}{dt}\right]_{acc}$$

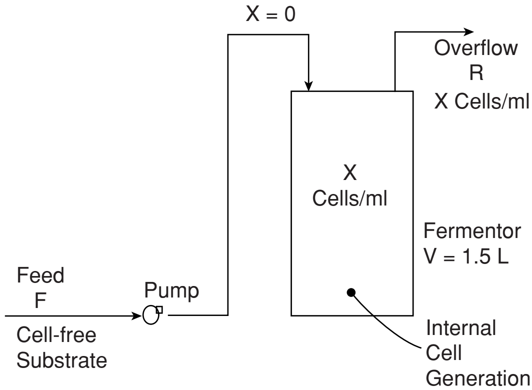

Figure 3.13 Material balance around a fermentor.

If cell mass is constant, then the last term on the right is zero. Cell growth is often expressed in terms of a generation time, g, which is the average time for doubling of cell numbers. Let t = time in hours. Then:

$$x = x_o(2)^{(t/g)}$$

Differentiating to obtain the generation term in the material balance equation:

$$\left[\frac{dx}{dt}\right]_{gen} = \frac{d}{dt}\left[x_0(2)^{(t/g)}\right] = x_0(g^{-1})(2)^{(t/g)}\ln 2 = xg^{-1}\ln 2$$

Substituting in the material balance equation and dropping out zero terms:

$$V \cdot x \cdot g^{-1}\ln 2 = R\,x;\ \text{x cancels out on both sides.}$$
$$R = V(\ln 2)g^{-1}$$

Substituting known quantities:

$$R = \frac{1.5\ln 2}{1.5} = \frac{0.693\ L}{h}$$

In continuous fermentation, the ratio R/V is the dilution rate, and when the fermentor is at a steady state in terms of cell mass, the dilution rate must equal the specific growth rate. The specific growth rate is the quotient: rate of growth/cell mass $= (1/x)(dx/dt)_{gen} = \ln 2/g$. Thus, the dilution rate to achieve steady state is strictly a function of the rate of growth of the organism and is independent of the cell mass present in the fermentor.

(b) When the feed rate is reduced, the cell number will be in an unsteady state. The material balance will include the accumulation term. Substituting the expression for cell generation into the original material balance equation with accumulation:

$$V\frac{x\ln 2}{g} = Rx + V\frac{dx}{dt}$$

$$x\left[\frac{V\ln 2}{g} - R\right] = v\frac{dx}{dt}$$

Separating variables and integrating:

$$\frac{dx}{x} = \left[\frac{\ln 2}{g} - \frac{R}{V}\right]dt$$

$$\ln\ x = \left[\frac{\ln 2}{g} - \frac{R}{V}\right]t + C$$

At $t = 0$, $x = 10,000$ cells/mL; $C = \ln(10,000)$.

$$\ln\left(\frac{x}{10,000}\right) = \left[\frac{\ln 2}{g} - \frac{R}{V}\right]t$$

Substituting: $V = 1.5$ L; $R = 0.8(0.693\ L/h) = 0.554\ L/h$; $g = 1.5$ h; $t = 10$ h

$$\ln\left(\frac{x}{10,000}\right) = 0.927;\ x = 10,000(2.5269)$$

$$x = 25,269\ \text{cells/ml}$$

3.3 BLENDING OF FOOD INGREDIENTS

3.3.1 Total Mass and Component Balances

These problems involve setting up total mass and component balances and involve simultaneously solving several equations.

Example 3.13. Determine the amount of a juice concentrate containing 65% solids and single-strength juice containing 15% solids that must be mixed to produce 100 kg of a concentrate containing 45% solids.

Solution:

The diagram for the process is shown in Fig. 3.14.

Total mass balance: $X + Y = 100$; $X = 100 - Y$
Solid balance: $0.65X + 0.15Y = 100(0.45) = 45$
Substituting $(100 - Y)$ for X:

$$0.65(100 - Y) + 0.15Y = 45$$

$$65 - 0.65Y + 0.15Y = 45$$

$$65 - 45 = 0.65Y - 0.15Y$$

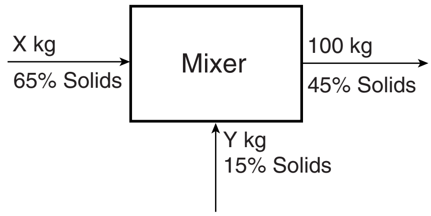

Figure 3.14 Material flow and composition in a process for blending of juice concentrates.

$20 = 0.5Y$

$Y = 40$ kg single strength juice

$X = 60$ kg 65% concentrate

Example 3.14. Determine the amounts of lean beef, pork fat, and water that must be used to make 100 kg of a frankfurter formulation. The compositions of the raw materials and the formulations are

Lean beef: 14% fat, 67% water, 19% protein.
Pork fat: 89% fat, 8% water, 3% protein.
Frankfurter: 20% fat, 15% protein, 65% water.

Solution:

The diagram representing the various mixtures being blended is shown in Fig. 3.15.

Total mass balance: $Z + X + Y = 100$
Fat balance: $0.14X + 0.89Y = 20$
Protein balance: $0.19X + 0.03Y = 15$

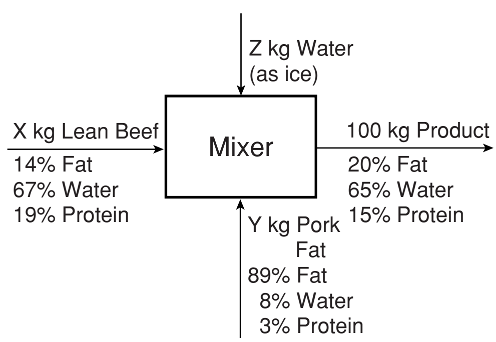

Figure 3.15 Composition and material flow in blending of meats for a frankfurter formulation.