Silo design

Lecture 15

Silo Design

- The agricultural granular product like cereals which are mostly stored in silo, constitute a friable cohesion less mass.
- The equilibrium granular mass depends upon the
  - speed of filling,
  - the ways the grains are packed together
  - the height of fall of grain in silo while loading
  - the compressibility of the material.
- The above factors affect the characteristics of grain like –
  - bulk density
  - internal angle of friction,
  - coefficient of friction on the walls.

Silo Design

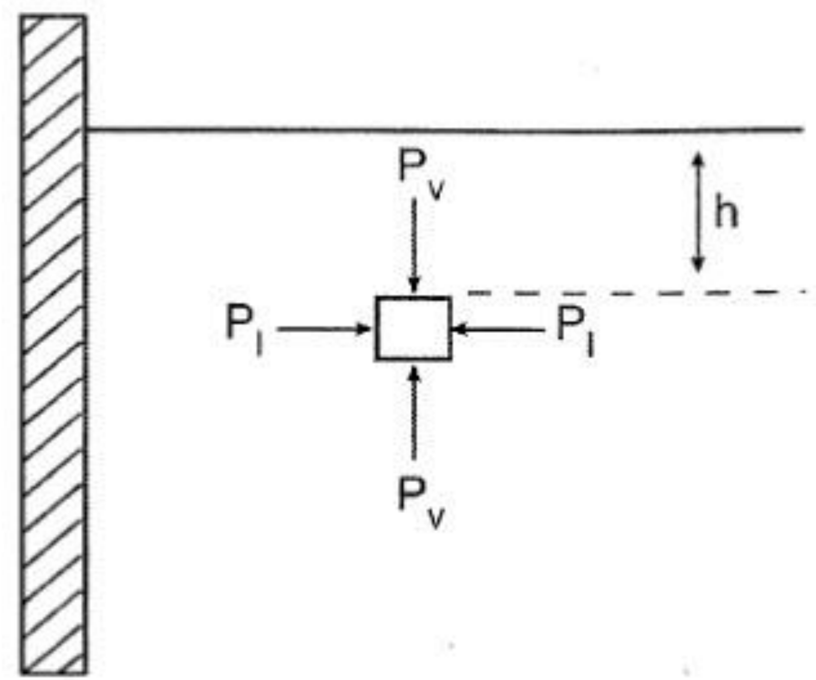
- The equilibrium laws for grains are not well developed. It does not fit in the theory of elasticity.
- Therefore, it is very difficult to directly determine the pressure acting on the walls. The walls of the silos have to sustain the **self weight** and **loads** during construction.
- The silos walls are subjected to two main forces or thrusts, (1) the horizontal thrust due to loading of material which tries to open up the silo and (2) the vertical pressure which is generated due to loading of material on the walls.

Effect of speed of filling

- The speed of filling of granular material affects the stress development on silo.
- When the material is filled **very rapidly** without any compaction other than the compaction produced in the operation, the **greatest forces** are obtained for the **lateral thrust** while the **apparent density and the angle of internal** friction of the material are at **minimum values**.
- On the contrary, when the grain is filled at such a **slow rate** which provides every possibility of compaction, the **greatest forces** are obtained for the **vertical pressure**. In this case the **bulk density and the angle of internal friction** of material are at **maximum values**.

Effect of silo wall on pressures

- The characteristics values of the granular material which cause greatest stresses should be taken into account while determining the lateral and vertical pressures of storage silo.



Effect of silo wall on pressures

- These values are function of the silo wall nature.
  - For **smooth** walled silo, the **greatest vertical and lateral pressure** be estimated with **maximum values of bulk density and angle of internal friction** while the **minimum value of angle of friction** of grains on the walls be taken.
  - For **rough** walls, the **lateral pressure** should be calculated with values of **maximum bulk density, minimum angle of internal friction** and the **minimum angle of friction of the grains** on the walls. The greatest **vertical pressure** should be estimated with **maximum bulk density, maximum angle of internal friction**, and the **minimum angle of friction of the grains on the walls**.

Lateral Pressure calculation

- The grain pressure in bins were first calculated as being a semi liquid of same density as the grains.
- The lateral pressure was first calculated using the hydrostatic formula.

$$\mathbf{P_L = w \times h}$$

Where, P_L = lateral pressure exerted by the grain on the bin wall

w = density of grain

h = depth of grain from the top of the bin surface

- This formula has **serious deficiencies** because many structures buckled under the vertical load arising from the friction of the grain on the walls.

Rankine’s theory

- The formula was modified to incorporate by multiplying a factor 'K ' known as Rankine's earth pressure coefficient.

$$P_l = K\,w\,h$$

Where K = Rankine’s coefficient

$$= \frac{1 - \sin\phi}{1 + \sin\phi}$$

ϕ = angle of internal friction of the grain

Rankine's theory

- The Rankine's formula is based on the following principle, the resistance to displacement by sliding along a given plane in a loose granular mass is equal to the normal pressure exerted between the parts of the mass on either side of the plane, multiplied by the specific constant.

The internal angle of friction of the grain was assumed by theoreticians and engineers, as equal to the natural angle of repose.

Therefore, $$P_1 = wh\frac{1-\sin\phi}{1+\sin\phi} \qquad \text{...4.12}$$

Rankine's theory

- For the Rankine theory Mohr circle may be constructed. The angle 'ϕ' is the angle of internal friction of grain. P_v and P_L are the maximum and minimum principal stresses for which the Mohr circle is constructed.

Mohr Circle for Rankine's theory

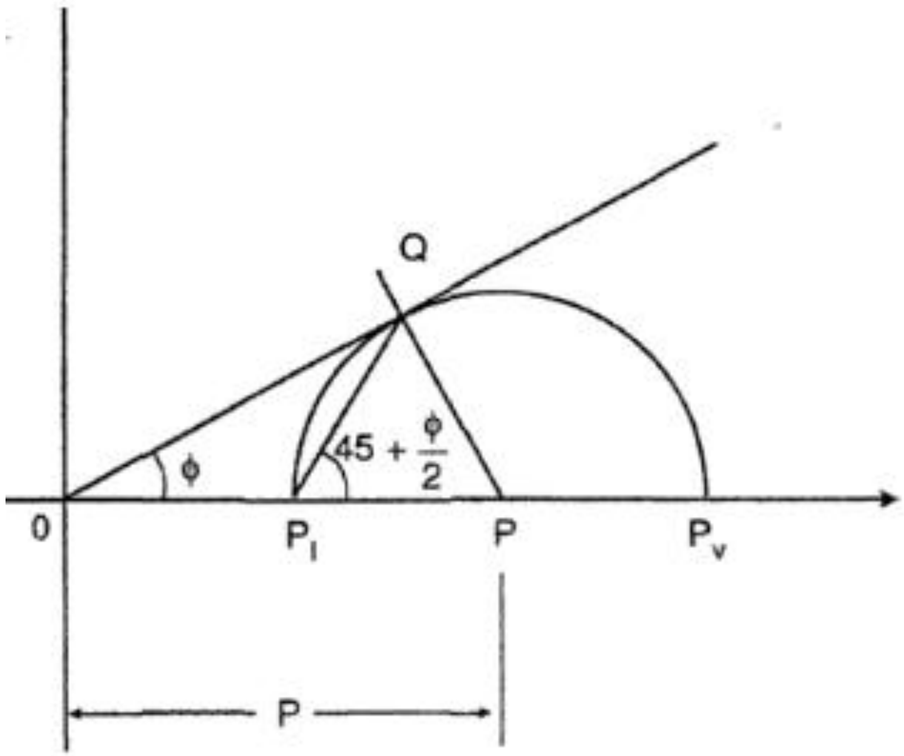


Fig. 4.18 : Mohr circle for Rankine theory

Now $$\sin\phi = \frac{QP}{OP} = \frac{\frac{1}{2}(P_v - P_l)}{\frac{1}{2}(P_v + P_l)}$$

$\therefore$ $$P_v \sin\phi + P_l \sin\phi = P_v - P_l$$

or, $$P_v(1 - \sin\phi) = P_l(1 + \sin\phi)$$

or, $$P_l = \frac{1 - \sin\phi}{1 + \sin\phi} P_v$$

Mohr Circle for Rankine's theory

or, $P_l = K P_v$

where, K = Rankine coefficient = Coefficient of earth pressure

$$K = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45 - \frac{\phi}{2}\right)$$

The Rankine formula is used for determination of pressure induced by granular materials against retaining wall in shallow bins.

Airy theory

Airy developed a theory for pressure induced by granular materials against retaining wall or in shallow bins. The lateral pressure exerted by grains in a shallow bin can be given by Airy equation

$$P_l = w\,h\left[\frac{1}{\sqrt{\mu(\mu+\mu')}+\sqrt{(1+\mu^2)}}\right]^2 \qquad \ldots 4.13$$

where, w = grain bulk density

h = depth of grain to point under consideration

μ = coefficient of friction of grain on grain

= $\tan\phi$, ϕ is angle of internal friction

$\mu' = \tan\phi'$, ϕ' is the angle of wall friction.

Problem : Wheat weighing 900 kg/m^3 is loaded in a circular concrete silo of 3 m internal diameter and a clear height of 8 m. The angle of internal friction for wheat is 25° and that for wheat and concrete is 24°.

Applying Airy theory, calculate the maximum lateral pressure at the bottom of bin section. (ARS, 1984)

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Applying Airy theory, calculate the maximum lateral pressure at the bottom of bin section. (ARS, 1984)

Solution : Airy formula for calculating lateral pressure against a shallow bin wall is:

$$L = w\,h\left[\frac{1}{\sqrt{\mu\,(\mu+\mu')}+\sqrt{1+\mu^2}}\right]^2$$

where,

L = lateral pressure

W = bulk density of grain

h = depth of bin (point) under consideration

μ = coefficient of friction of grain on grain

$= \tan\,\phi$, ϕ is the angle of internal friction

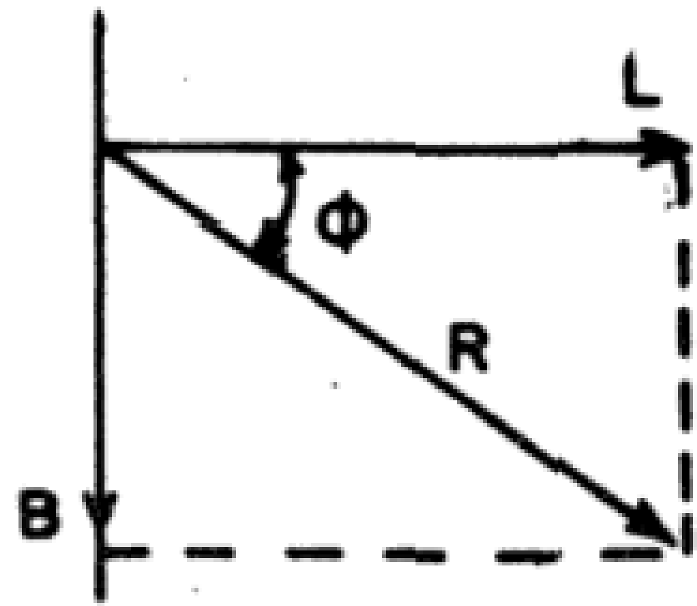
$\mu' = \tan\,\phi'$, ϕ' is the angle of wall friction

$$L = 900\times 8\left[\frac{1}{\sqrt{0.466\,(0.466+0.445)}+\sqrt{1+0.217}}\right]^2$$

$$= 900\times 8\left[\frac{1}{0.6515+1.1031}\right]^2$$

$$= 2338.5\ kg/m^2$$

Pressure distribution in bin



R = Resultant pressure produced by friction of the grain against the bin wall
L = Lateral pressure on the wall
B= Pressure at the bottom
ϕ = Angle of external friction

$$L = R \cos \phi$$
$$B = R \sin \phi$$
$$B = L \tan \phi$$

Pressure distribution in bin

- At a particular depth inside the silo, the load on the bottom, or the **total vertical pressure** is equal to the difference between the total weight of the stored grain and the total load balanced by the friction of the material on the walls.
- The **vertical pressure** on the bin bottom increases with the level of filling upto certain value, beyond this level of filling, the vertical pressure attains a maximum value and the pressure becomes constant.

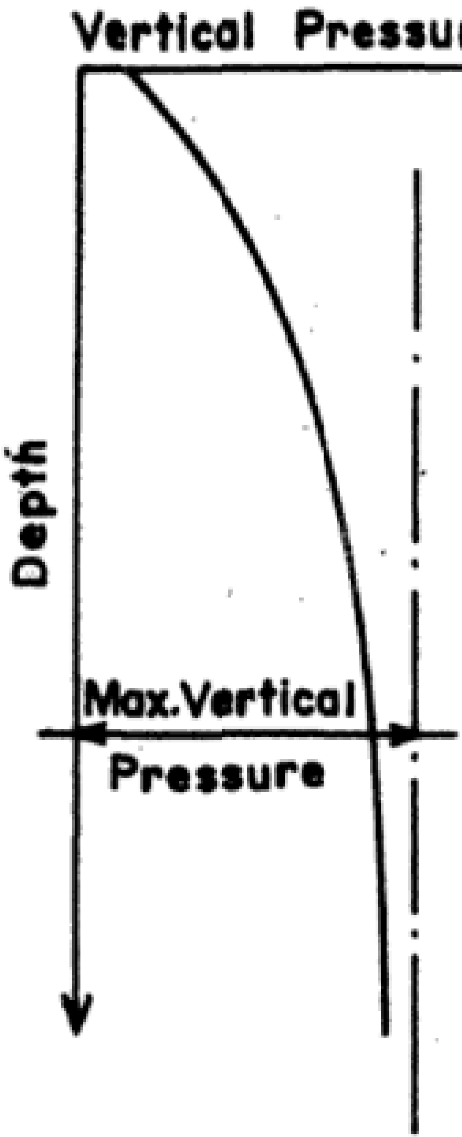


Fig. 4.20 a : Vertical pressure in a bin

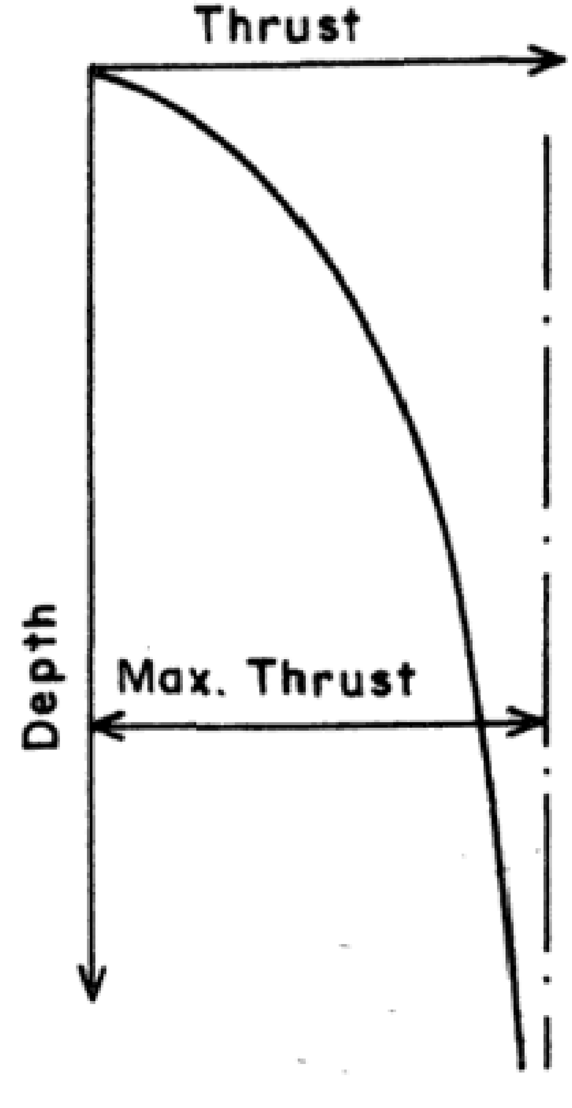


Fig. 4.20 b : Lateral pressure on a bin wall

Pressure distribution in conical bottom

In a conical bottom vertical bin, at the **transition point** of the discharge hopper, the pressure reduces. The pressure again starts building up in the direction of outlet.

This shows that the pressure in the discharge hopper is not dependent on the height of filling of grain in the bin.

Due to such phenomenon of pressure, the flow pattern of grain from a vertical bin with gravitational discharge is influenced by the level of filling in the bin.

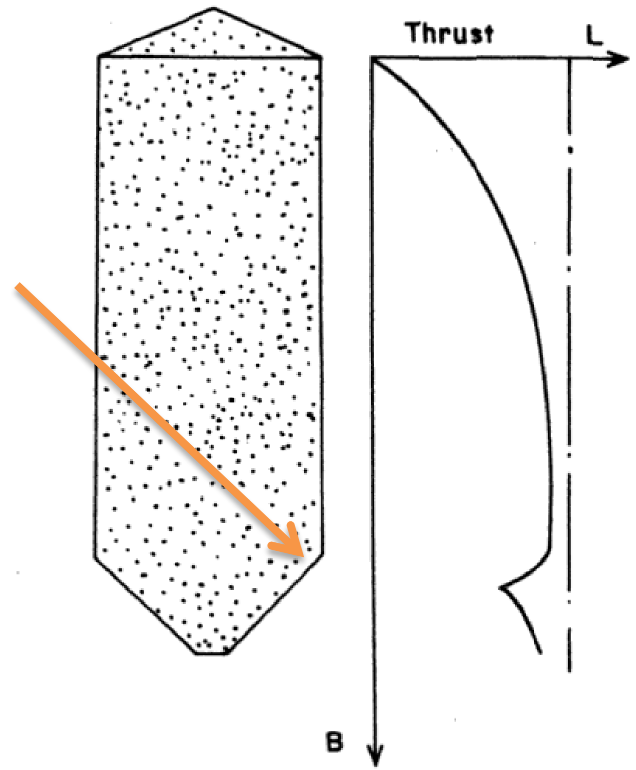


Fig. 4.21 : Lateral pressure distribution in a conical bottom vertical bin

JANSEEN FORMULA

- ✓ Derived for pressure distribution in deep bins.
- ➢ Uses friction between the grain and bin wall.
- ➢ It is widely used for bins.
- ➢ Bin’s design is safe because of higher safety factor.

✓ Derivation

Let,

h = Depth of bin

μ = Coefficient of friction of the grain on the wall

K = Safety factor

P_V = Vertical Pressure

P_L = Lateral Pressure

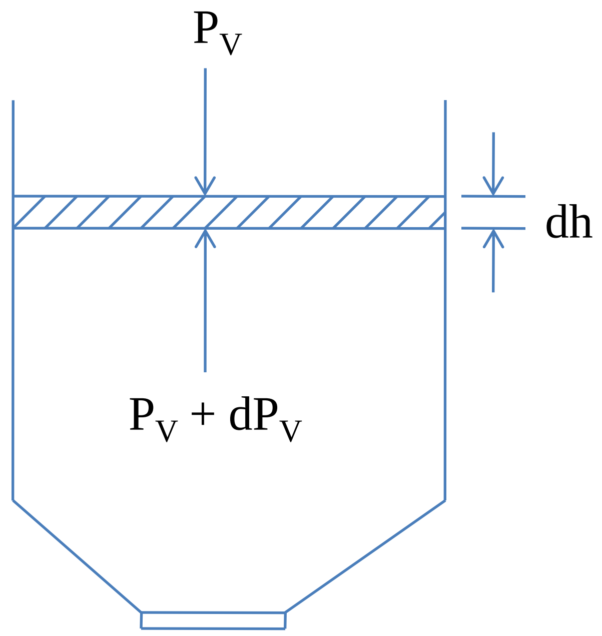
P = Total lateral wall load

w = Unit weight of grain

R = Hydraulic mean radius

r = Radius of small disc of height dh

π = Constant



Now,

During storage forces acting are given below :-

1. Vertical downward force :-

Due to mass of material = unit weight × volume of material

$$= w \times \pi r^2 dh$$

2. Vertical upward forces :-

(a). Effective Upward Force = Unit upward force × Cross-sectional area

$$= dP_V \times \pi r^2$$

(b). Due to friction factor grain and wall of grain

= Force due to friction × area of the bin surface

$$= \mu . P_L \times 2\pi r . dh$$

In case of equilibrium , all the downward forces will be equal to the upward forces,

So,

$$w \times \pi r^2 dh = (dP_V \times \pi r^2) + (\mu . P_L \times 2\pi r . dh)$$

Dividing by πr,

$$w \times r . dh = (r . dP_V) + (2 \mu P_L dh)$$

Putting (dh) term on one side,

$$(w \times r.dh) - (2\mu P_L dh) = r.dP_V$$

$$(wr - 2\mu P_L)\, dh = r.dP_V$$

$$dh = (r.dP_V) / (wr - 2\mu P_L)$$

$K = P_L/P_V$ and $R = r/2$ (hydraulic radius)

$$dh = (r.dP_V) / (wr - 2\mu K P_V)$$

Integrating the equation for the following limits:-

$h \Rightarrow (0, h)$

$P_V \Rightarrow (0, P_V)$

$$\int_0^h dh = \int_0^{P_v} \frac{r}{wr - 2\mu K P_v}\, dP_v$$

Let , $x = (wr - 2\mu K P_V)$

$dx = - 2\mu K . dP_V$

$$dPv = -\frac{dx}{2\mu K}$$

at $P_V = 0$, $x = wr$

at $Pv = P_V$, $x = (wr - 2\mu K P_V)$

$$\int_0^h dh = r \int_{wr}^{(wr - 2\mu K P_v)} \frac{1}{x} \frac{dx}{-2\mu K}$$

$$= \frac{-r}{2\mu K} \int_{wr}^{wr - 2\mu K P v} \frac{1}{x} dx$$

$$= \frac{-r}{2\mu K} [\log_e (wr - 2\mu K P v) - \log_e wr]$$

$$h = -\frac{r}{2\mu K}\log_e\left[\frac{wr - 2\mu KPv}{wr}\right]$$

$$h = \frac{-r}{2\mu K}\log_e\left(1 - \frac{2\mu KPv}{wr}\right)$$

$$-\frac{2\mu Kh}{r} = \log_e\left(1 - \frac{2\mu KPv}{wr}\right)$$

Putting, R = r/2

$$-\frac{2\mu Kh}{r} = \log_e\left(1 - \frac{\mu KPv}{wR}\right)$$

$$e^{-\left(\frac{\mu kh}{R}\right)} = 1 - \frac{\mu KPv}{wR}$$

$$\frac{\mu KPv}{wR} = 1 - e^{-\left(\frac{\mu Kh}{R}\right)}$$

$$K.Pv = \frac{wR}{\mu}\left[1 - e^{-\frac{\mu K h}{R}}\right]$$

Putting , P_L = K Pv

$$Pl = \frac{wR}{\mu}\left[1 - e^{-\frac{\mu K h}{R}}\right]$$

The maximum lateral pressure of a bin can be given by the following expression,

$$P_{max} = \frac{w\,R}{\mu} \qquad \ldots 4.22$$

Problem : A R.C.C cylindrical grain storage bin has internal diameter of 5 m and is 20 m deep. It is completely filled with paddy weighing 600 kg/m^3. The angle of internal friction for paddy can be taken as 35° while the angle of friction between paddy and bin wall is 30°. The ratio of horizontal and vertical pressure intensity, k, is 0.4. Calculate the lateral pressure intensity at 2.0 m interval. (ARS 1990)

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Solution : The lateral pressure by Janssen theory,

$$P_l = \frac{wR}{\mu}\left[1 - e^{-\frac{\mu K h}{R}}\right]$$

Given that,

$$w = 600\ \text{kg/m}^3$$

$$R = \frac{D}{4} = 1.25\ \text{m}$$

$$\mu = \tan\ 30° = 0.577$$

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$$\mu = \tan 30° = 0.577$$

$$K = \frac{P_l}{P_v} = 0.4$$

Substituting the values of various depths in above equation, we get

$$P_l \text{ at } 2.0 \text{ m} = \frac{600 \times 1.25}{0.577}\left[1 - e^{-\frac{0.577 \times 0.4 \times 2}{1.25}}\right]$$

$$= 401.6 \text{ kg/m}^2$$

$$P_l \text{ at } 4.0 \text{ m} = \frac{600 \times 1.25}{0.577}\left[1 - e^{-\frac{0.577 \times 0.4 \times 4}{1.25}}\right]$$

$$= 679.8 \text{ kg/m}^2$$

The values of lateral pressure at different depths are given in the following table.

depth, m	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0
P_l, kg/m^2	870.8	1003	1094.4	1158	1202	1232.2	1253	1268.6

Problem : A cylindrical silo of 2.5 m diameter and 20 m in height, is filled with wheat. Calculate the load on the bottom, and the lateral thrust at every 2.0 m depth on the walls. The silo is made of steel with smooth walls. The characteristics of stored wheat are as follows.

Minimum bulk density : 720 kg/m^3

Maximum bulk density : 830 kg/m^3

Minimum angle of internal friction : 25°

Maximum angle of internal friction : 30°

Minimum angle of friction on smooth sheeting : 18°

Angle of repose : 25°

For **smooth** walled silo, the **greatest vertical and lateral pressure** be estimated with **maximum values of bulk density** while the **minimum value of angle of internal friction and angle of friction** of grains on the walls be taken.

Variation of P_v/P_L with depth of bin

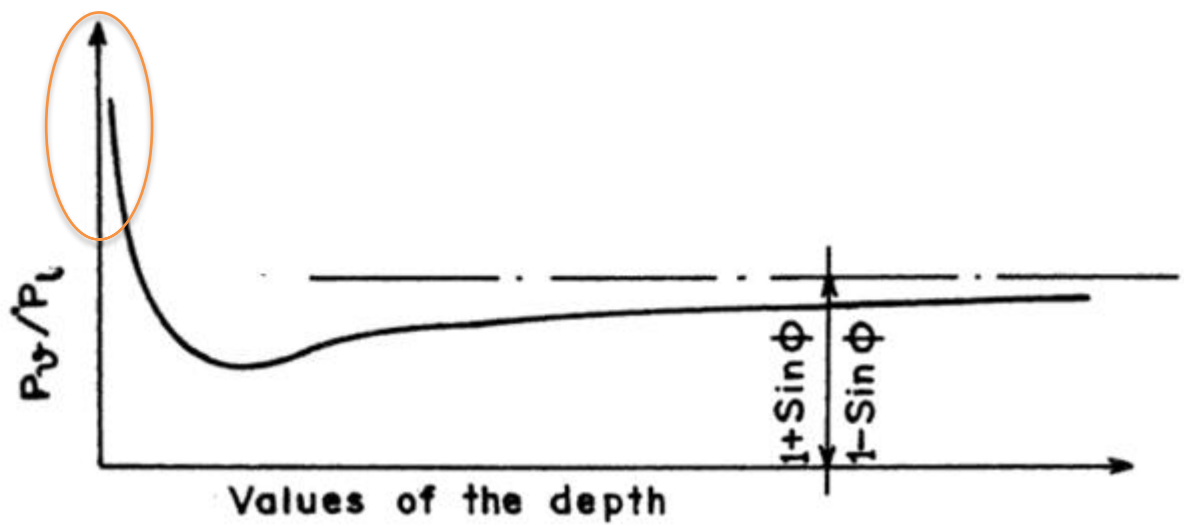
In (Mohr circle) the mean of the stresses is denoted by 'P', then radius of Mohr's circle will be $P \sin \phi$, therefore

$$P_l = P(1 - \sin \phi)$$

$$P_v = P(1 + \sin \phi)$$

or,

$$\frac{P_l}{P_v} = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$



Plot of ratio of vertical to lateral pressures Vs depth of bín

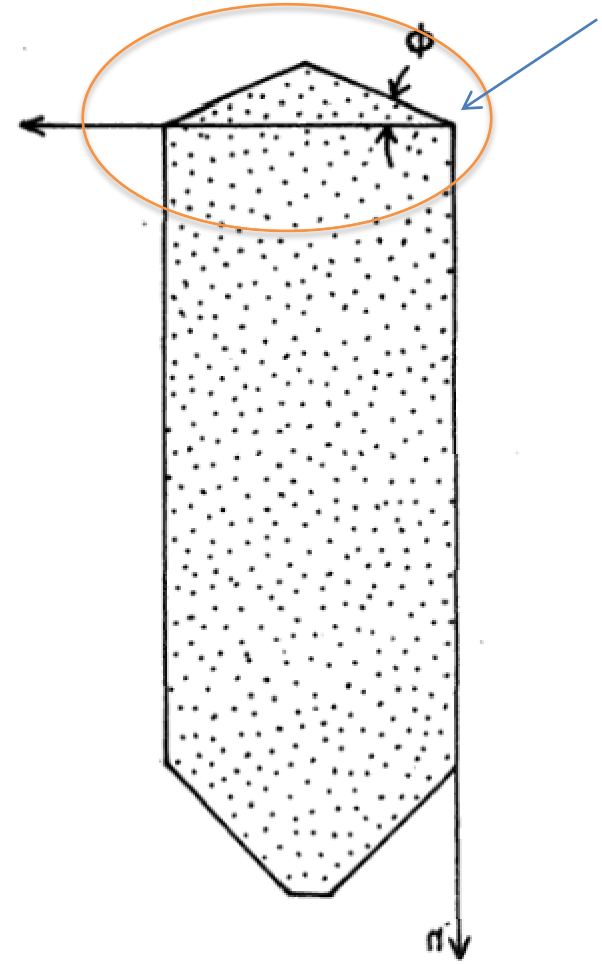
Variation of P_V/P_L with depth of bin

At the origin of the abscissae, the lateral pressure is zero, but the mean vertical pressure has some finite value. This value is equal to the weight of the upper cone of material divided by the cross-sectional area of the bin (Fig. 4.24). The height of the upper cone of stored material is given by;

$$Z = r \tan \phi \qquad \ldots 4.20$$

where, r = radius of cylindrical bin
= radius of the circle inscribed in the polygonal outline of the cross-section.

Therefore, the value of $\frac{P_v}{P_l} \to \infty$ and the curve representing the ratio $\frac{P_v}{P_l}$ tends towards infinity having the axis of the ordinates as asymptote.



Characteristic abscissa

- Reimberts have also defined a characteristic abscissa as a function of the angle of internal friction ϕ, and angle of friction ϕ’ of the grains on the bin wall. The value of characteristic abscissa 'A' can be given by the following expression.

$$A = \frac{R}{\mu K}$$

Characteristic abscissa

The characteristic abscissa in the silo, for the calculation of large wall can be given by the following equation

$$A = \frac{D}{4 \tan \phi' \tan^2 \left(\frac{\pi}{4} - \frac{\phi}{2} \right)} - \frac{Z}{3} \qquad \ldots 4.21$$

for small walls, $D = \frac{4a}{\pi}$

where, a = internal length of side of a silo of square or polygonal shape, or the shorter side of a silo of rectangular cross-section.

ϕ' = angle of friction of the stored grain on the wall of the silo.

ϕ = natural angle of repose of stored grain

D = internal diameter of a cylindrical silo, or diameter of the inscribed circle of a polygonal silo having more than 4 sides.

The lateral pressure at various depth of the silo is a function of depth and the coefficient of friction and can be given by,

$$P_h = \frac{F(h)}{L \cdot \mu}$$

The values of $F(h)$ further can be given by

$$F(h) = w \cdot s \left[1 - \left(\frac{h}{A} + 1 \right)^{-2} \right] \quad \ldots 4.24$$

or

$$P_h = \frac{w\,s}{L\,\mu} \left[1 - \left(\frac{h}{A} + 1 \right)^{-2} \right] \quad \ldots 4.25$$

The load of grain balanced by the friction on the walls of a *silo* is a function of depth of the silo and can be given by the following expression,

$$L_f = \frac{w \cdot s \cdot h^2}{h + A} \quad \ldots 4.23$$

where, L_f = load of grain balanced by the friction on the walls

w = bulk density of grain

s = cross–sectional area of silo

h = depth of silo

A = characteristic abscissa

or $$P_h = \frac{w \cdot s}{L \cdot \mu}\left[1 - \left(\frac{h}{A}+1\right)^{-2}\right] \qquad \text{...4.25}$$

Since $\frac{w \cdot R}{\mu}$ is equal to $P_{max,}$

$$P_h = P_{max}\left[1 - \left(\frac{h}{A}+1\right)^{-2}\right] \qquad \text{...4.26}$$

The total load on the bottom of the silo can be expressed by,

$$L = s \cdot q_z$$

where, $$q_z = w\left[h\left(\frac{h}{A}+1\right)^{-1} + \frac{Z}{3}\right] \qquad \text{...4.27}$$

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Angle of repose : 25°

Solution : Height of the upper cone of stored wheat,

$$Z = r \tan 25^\circ = \frac{2.5}{2} \times 0.4663$$

$$= 0.5825 \text{ m}$$

Mean hydraulic radius, $R = \frac{D}{4} = \frac{2.5}{4} = 0.625$

$$\tan 25^\circ = 0.4663$$

$$\tan 18^\circ = \mu = 0.3249$$

$$\tan^2\left(\frac{\pi}{4} - \frac{25^\circ}{2}\right) = 0.4058$$

Therefore the characteristic abscissa 'A'

$$A = \frac{R}{\mu K} = \frac{0.625}{0.3249 \times 0.4058}$$

$$= 4.74 \text{ m}$$

$$P_{max} = \frac{w_{max} R}{\mu} = \frac{830 \times 0.625}{0.3249}$$

$$= 1596.6 \text{ kg/m}^2$$

Load at the bottom,

height of grain against the wall, $h = 20 - 0.5825$

$$= 19.417 \text{ m}$$

$$L_{19.417} = 830 \left[19.417 \left(\frac{19.417}{4.74} + 1 \right)^{-1} + \frac{0.5825}{3} \right] \times \frac{\pi \times (2.5)^2}{4}$$

$$= 16314.8 \text{ kg}$$

Values of lateral pressure at every 2.0 m depth are given in the following table.

h, m	h/A	$1 - (h/A + 1)^{-2}$	P_h, Kg/m^2
2.0	0.4219	0.5054	806.91
4.0	0.844	0.7060	1127.05
6.0	1.266	0.8052	1285.66
8.0	1.687	0.8614	1375.46
10.0	2.109	0.8965	1431.42
12.0	2.531	0.9197	1468.54
14.0	2.953	0.9360	1494.42
16.0	3.375	0.9477	1513.18
18.0	3.797	0.9565	1527.21
19.417	4.096	0.9614	1535.12