The 6-hour unit graph derived as above is shown in Fig. 5.17.

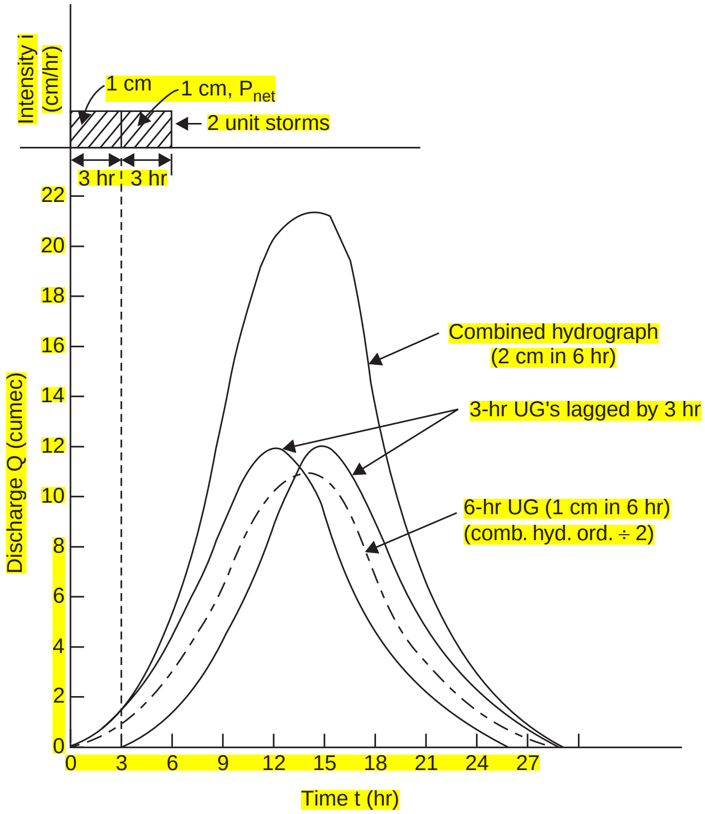

Fig. 5.17 Changing a short duration UG to a long multiple duration

Case (*ii*) *Changing a Long Duration Unit Hydrograph to a Shorter Duration by S-curve Technique*

5.6 *S*-CURVE METHOD

S-curve or the summation curve is the hydrograph of direct surface discharge that would result from a continuous succession of unit storms producing 1 cm in t_r–hr (Fig. 5.18). If the time base of the unit hydrograph is T hr, it reaches constant outflow (Q_e) at T hr, since 1 cm of net rain on the catchment is being supplied and removed every t_r hour and only T/t_r unit graphs are necessary to produce an *S*-curve and develop constant outflow given by,

$$Q_e = \frac{2.78\,A}{t_r} \qquad \text{...(5.6)}$$

where Q_e = constant outflow (cumec)

t_r = duration of the unit graph (hr)

A = area of the basin (km^2)

Given a t_r-hour unit graph, to derive a t_r'-hour unit graph ($t_r' \geq t_r$)—Shift the S-curve by the required duration t_r' along the time axis. The graphical difference between the ordinates of the two *S*-curves, *i.e.,* the shaded area in Fig. 5.18 represents the runoff due to t_r' hours rain at an intensity of $1/t_r$ cm/hr, *i.e.,* runoff of t_r'/t_r cm in t_r' hours. To obtain a runoff of 1 cm in t_r' hours (*i.e.,* t_r'-hour *UG*), multiply the ordinates of the *S*-curve difference by t_r/t_r'. This technique may be used to alter the duration of the given unit hydrograph to a shorter or longer duration. The longer duration need not necessarily be a multiple of short.

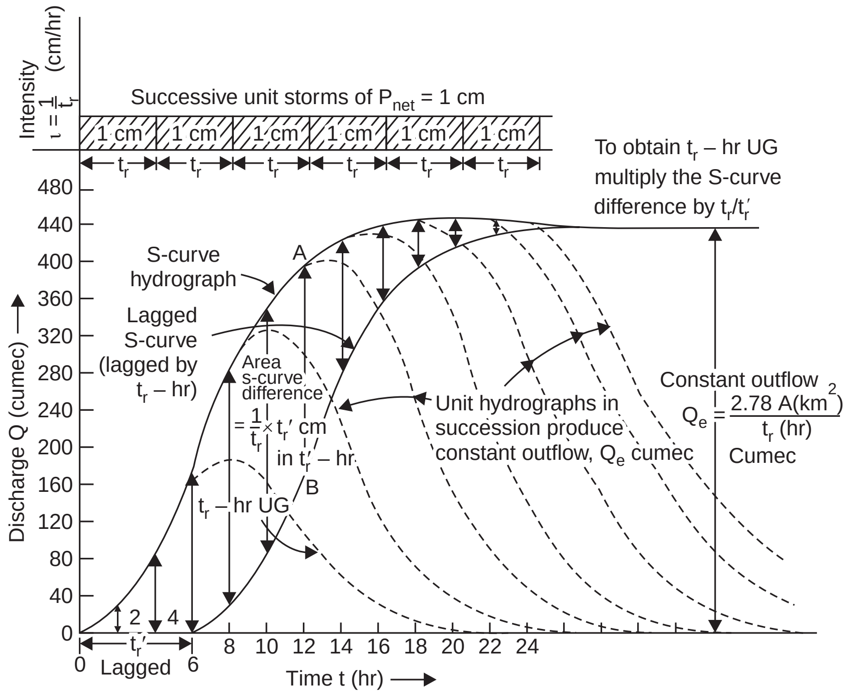

Fig. 5.18 Changing the duration of UG by S-curve technique (Example 5.5)

Example 5.4 *The ordinates of a 4-hour unit hydrograph for a particular basin are given below. Derive the ordinates of (i) the S-curve hydrograph, and (ii) the 2-hour unit hydrograph, and plot them, area of the basin is 630 km*^{2}.

Time (hr)	*Discharge (cumec)*	*Time (hr)*	*Discharge (cumec)*
0	*0*	*14*	*70*
2	*25*	*16*	*30*
4	*100*	*18*	*20*
6	*160*	*20*	*6*
8	*190*	*22*	*1.5*
10	*170*	*24*	*0*
12	*110*		

Solution See Table 5.4

Table 5.4 Derivation of the S-curve and 2-hour unit hydrographs. (Example 5.4)

Time (hr)	*4-hr UGO (cumec)*	*S-curve additions (cumec) (unit storms after every 4 hr = t_r)*					*S-curve ordinates (cumec) (2) + (3)*	*lagged S-curve (cumec)*	*S-curve difference (cumec) (4) – (5)*	*2-hr UGO (6) × 4/3 (cumec)*
1	*2*			*3*			*4*	*5*	*6*	*7*
0	0	—	—	—	—	—	0	—	0	0
2	25	—	—	—	—	—	25	0	25	50
4	100	0	—	—	—	—	100	25	75	150
6	160	25	—	—	—	—	185	100	85	170
8	190	100	0	—	—	—	290	185	105	210
10	170	160	25	—	—	—	335	290	65	130
12	110	190	100	0	—	—	400	355	45	90
14	70	170	160	25	—	—	425	400	25	50
16	30	110	190	100	0	—	430	425	5	10*
18	20	70	170	160	25	—	445	430	15	30
20	6	30	110	190	100	0	436	445	– 9	– 18*
22	1.5	20	70	170	160	25	446.5	436	10.5	21
24	0	6	30	110	190	100	436	446.6	– 10.5	– 21*

*Slight adjustment is required to the tail of the 2-hour unit hydrograph.

Col (5): lagged S-curve is the same as col (4) but lagged by $t_r' = 2$ hr.

Col (7): col (6) × $\frac{t_r}{t_r'}$, $t_r = 4$ hr. $t_r' = 2$ hr.

Col (3): No. of unit storms in succession = $T/t_r = 24/4 = 6$, to produce a constant outflow.

$Q_e = \frac{2.78\,A}{t_r} = \frac{2.78 \times 630}{4} = 437$ cumec, which agrees very well with the tabulated *S*-curve terminal value of 436. The *S*-curve additions can be written in one column, without having to write in 5 columns successively lagged by 4 hours (= t_r), as is illustrated in the example 5.5.

Plot col (4) versus col (1) to get the *S*-curve hydrograph, and col (7) versus col (1) to get the 2-hour unit hydrograph, as shown in Fig. 5.19.

Example 5.5 *The ordinates of a 4-hour unit hydrograph for a particular basin are given below. Determine the ordinates of the S-curve hydrograph and therefrom the ordinates of the 6-hour unit hydrograph.*

Time (hr)	*4-hr UGO (cumec)*	*Time (hr)*	*4-hr UGO (cumec)*
0	0	12	110
2	25	14	70
4	100	16	30
6	160	18	20
8	190	20	6
10	170	22	1.5
		24	0

Solution See Table 5.5.

Table 5.5 Derivation of the 6–hour UG for the basin (Example 5.5)

Time (hr)	*4-hour UGO (cumec)*	*S-curve[1] additions (cumec)*	*S-curve ordinates (cumec) (2) + (3)*	*lagged[2] C-curve (cumec)*	*S-curve difference (cumec) (4) – (5)*	*6-hr UGO (cumec) (6) × 4/6*
1	*2*	*3*	*4*	*5*	*6*	*7*
0	0 →	—	0	—	0	0
2	25 →	—	25	—	25	16.7
4	100 + →	0	100	—	100	66.7
6	160 + →	25	185	0	185	123.3
8	190	100	290	25	265	176.7
10	170	185	355	100	255	170.0
12	110	290	400	185	215	143.3
14	70	355	425	290	135	90.0
16	30	400	430	355	75	50.0
18	20	425	445	400	45	30.0
20	6	430	436	425	11	7.3
22	1.5	445	446.6	430	16.5	11.0
24	0	436	436	445	– 9	– 6.0

1—Start the operation shown with 0 cumec after t_r = 4 hr.

2—Lag the S–curve ordinates by t_r' = 6 hr.

Plot col (4) vs. col (1) to get S-curve hydrograph and col (7) vs. col (1) to get 6-hr unit hydrograph as shown in Fig. 5.19.

5.7 BERNARD'S DISTRIBUTION GRAPH

The distribution graph, introduced by Bernard in 1935, shows the percentages of total unit hydrograph, which occur during successive, arbitrarily chosen, uniform time increments, Fig. 5.20. It is an important concept of the unit hydrograph theory that all unit storms, regardless of their intensity, produce nearly identical distribution graphs.

The procedure of deriving the distribution graph is first to separate the base flow from the total runoff; the surface runoff obtained is divided into convenient time units, and the average rate of surface runoff during each interval is determined.

If the rainfall-runoff data for a short duration fairly uniform storm is known, the duration of net rainfall is taken as the unit period and the distribution percentages are computed directly. But if there are multiple storms of different intensities producing different net rains during successive unit periods, a trial and error procedure of applying the distribution percentages is followed, till the direct surface runoff during successive time intervals corresponds to the computed values. Once a distribution graph is derived for a drainage basin, any expected volume of surface runoff from the basin can be converted into a discharge hydrograph. By drawing a smooth curve along the steps of the distribution graph to give equal areas, a unit hydrograph may be obtained as shown in Fig. 5.20.

Example 5.6 *Analysis of the runoff records for a one day unit storm over a basin yields the following data*:

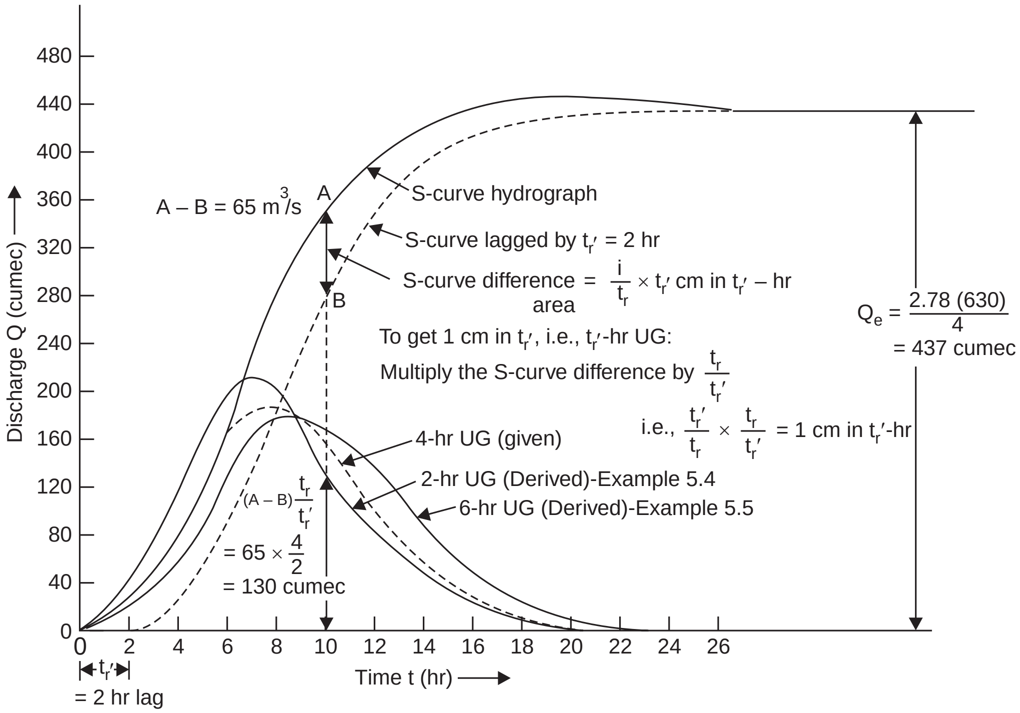

Fig. 5.19 Derivation of 2-hr & 6-hr UG from a 4-hr UG (Example 5.4 & 5.5)

Total stream flow at concentration point on successive days are 19.6, 62.4, 151.3, 133.0, 89.5, 63.1, 43.5, 28.6, and 19.6 cumec.

Estimated base flow during the corresponding period on successive days are 19.6, 22.4, 25.3, 28.0, 28.0, 27.5, 25.6, 22.5 and 19.6 cumec.

Determine the distribution graph percentages.

On the same basin (area = 2850 km^2) there was rainfall of 7 cm/day on July 15 and 10 cm/day on July 18 of a certain year. Assuming an average storm loss of 2 cm/day, estimate the value of peak surface runoff in cumec and the date of its occurrence.

Table 5.6 Derivation of distribution percentages

Day since beginning of direct runoff	*DRO on mid-day (cumec)*	*Percentage of ΣDRO*	*Remarks*
1	*2*	*3*	*4*
1	18	4.5	$= \frac{18}{402} \times 100$
2	96	24	
3	120	30	
4	82	20	
5	47	12	
6	25	6	
7	12	3	
8	2	0.5	
8 equal time (*i.e.*, a day) intervals	Σ *DRO* = 402	Total = 100.0	

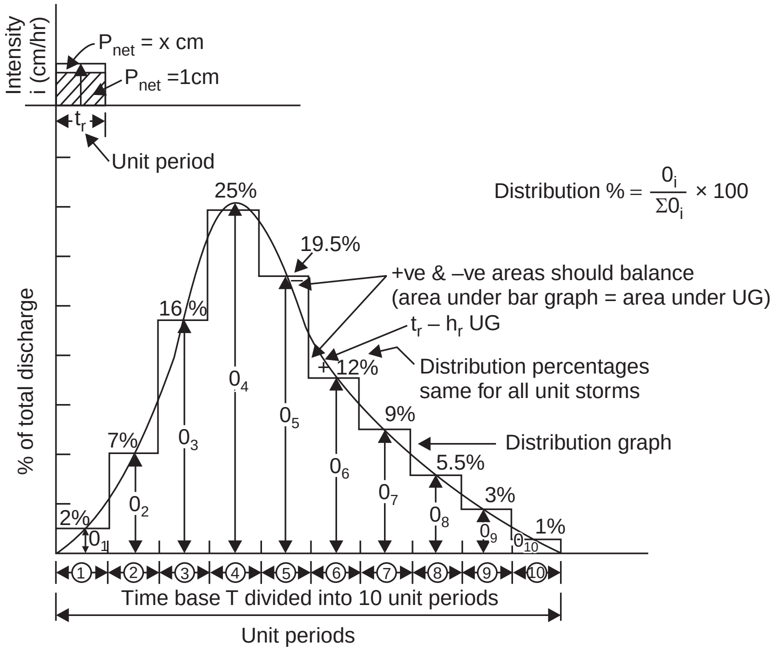

Fig. 5.20 Distribution graph (after Bernard, 1935)

Solution The total runoff hydrograph and estimated base flow are drawn in Fig. 5.21 and the direct runoff ordiantes on successive mid-days are determined as *DRO* = *TRO* – *BFO* and the percentages of direct runoff on successive days computed in Table 5.6. Column (3) gives the distribution percentages, and the derived distribution graph for 1-day unit storms is shown in Fig. 5.22.

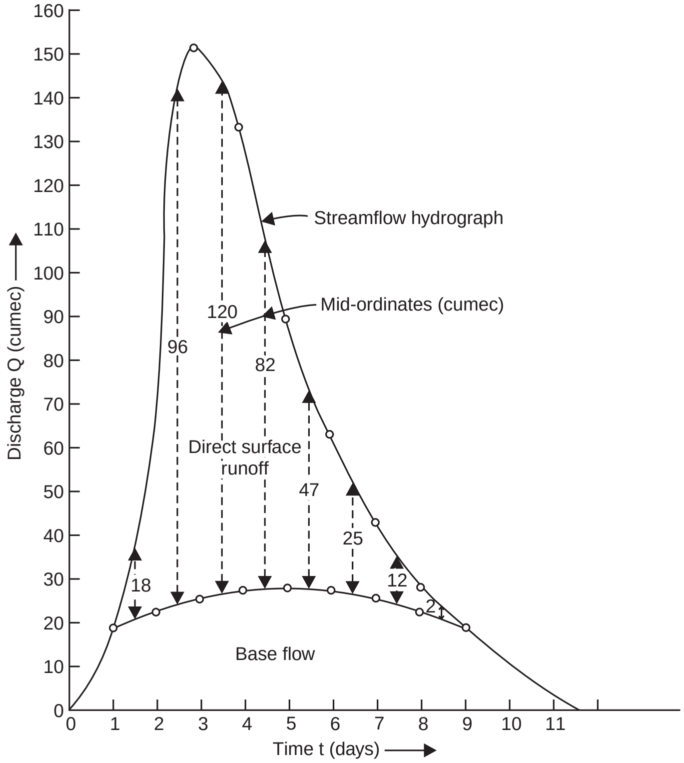

Fig. 5.21 Stream flow hydrograph (Example 5.6)

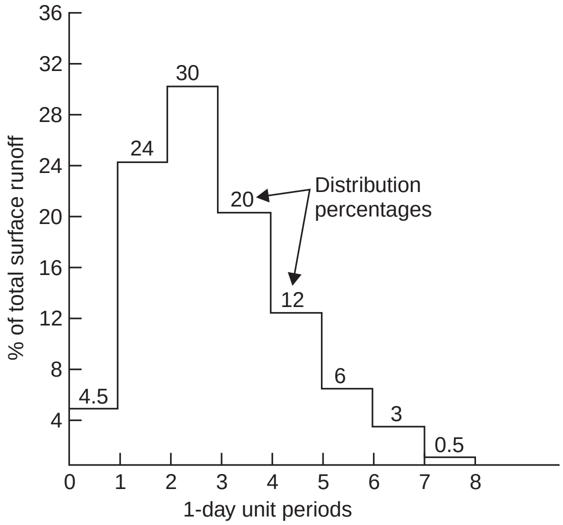

Fig. 5.22 Derived distribution graph (Example 5.6)

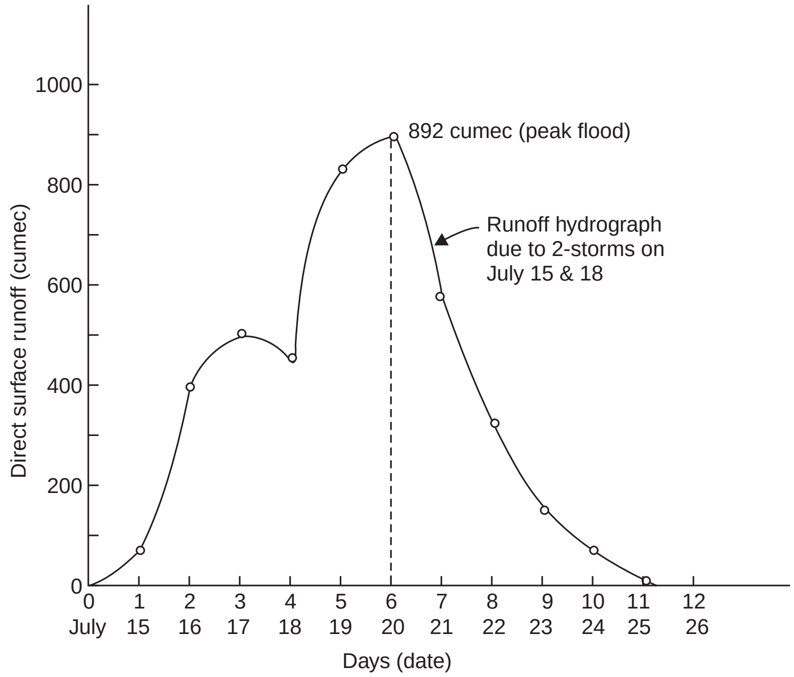

Fig. 5.23 Runoff Hydrograph (Example 5.6)

Applying the distribution percentages computed in col. (3) above the direct surface discharge on successive days due to the two storms (lagged by 3 days) is computed in Table 5.7

$$1 \text{ cm/day} = \frac{1}{100} \times \frac{2850 \times 10^6}{24 \times 60 \times 60} = 330 \text{ cumec}$$

The peak surface runoff is 892 cumec and occurs on July 20 of the year. The flood hydrograph is shown in Fig. 5.23.

Example 5.7 *Analysis of rainfall and runoff records for a certain storm over a basin (of area 3210 km^2) gave the following data:*

Rainfall for successive 2 hr periods: 2.5, 6.5 and 4.5 cm/hr.

An average loss of 1.5 cm/hr can be assumed.

Direct surface discharge at the concentration point for successive 2-hr periods: 446, 4015, 1382, 25000, 20520, 10260, 4900 and 1338 cumec.

Derive the unit hydrograph in the form of distribution percentages on the basis 2-hr unit periods.

Solution The rainfall may be considered for three unit periods of 2 hr each, then from Fig. 5.24,

$$T_{DSR} = t_R + T_r \quad \text{...(5.7)}$$

$$T = t_r + T_r \quad \text{...(5.7 } a\text{)}$$

$$\therefore \quad T = t_r + T_{DSR} - t_R$$

$$= 2 + 8 \times 2 - 3 \times 2 \quad \text{...(5.7 } b\text{)}$$

$$\therefore \quad T = \mathbf{12 \ hr}$$

Table 5.7 Application of distribution percentages to compute the direct surface discharge (Example 5.6)

		Unit periods (day)										
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
Total rainfall (cm) →		7			10							
Loss of rain (cm) →		2			2							
Net rain (cm) →		5			8	(= 13 cm)						
Unit distribution					Distribution (cm/day)							
Periods	Percentages											
1	4.5	0.225			0.36							
2	24		1.20			1.92						
3	30			1.50			2.40					
4	20				1.00			1.60				
5	12					0.60			0.96			
6	6						0.30			0.48		
7	3							0.15			0.24	
8	0.5								0.025			0.04
Total (cm/day):		0.225	1.20	1.50	1.36	2.52	2.70	1.75	0.985	0.48	0.24	0.04 = 13 cm (check)
× 330 = cumec:		74	396	495	450	833	$\boxed{892}$	578	325	158	79	13 (0.225 × 330) = 74 cumec)
Date: July		15	16	17	18	19	20	21	22	23	24	25

The base width is 12 hr or 6 unit periods. As a first trial, try a set of six distribution percentages of 10, 20, 40, 15, 10, 5 which total 100%. The direct surface discharge can be converted into cm/hr as

$$1 \text{ cm/hr} = \frac{1}{100} \times \frac{3210 \times 10^6}{60 \times 60} = 8920 \text{ cumec}$$

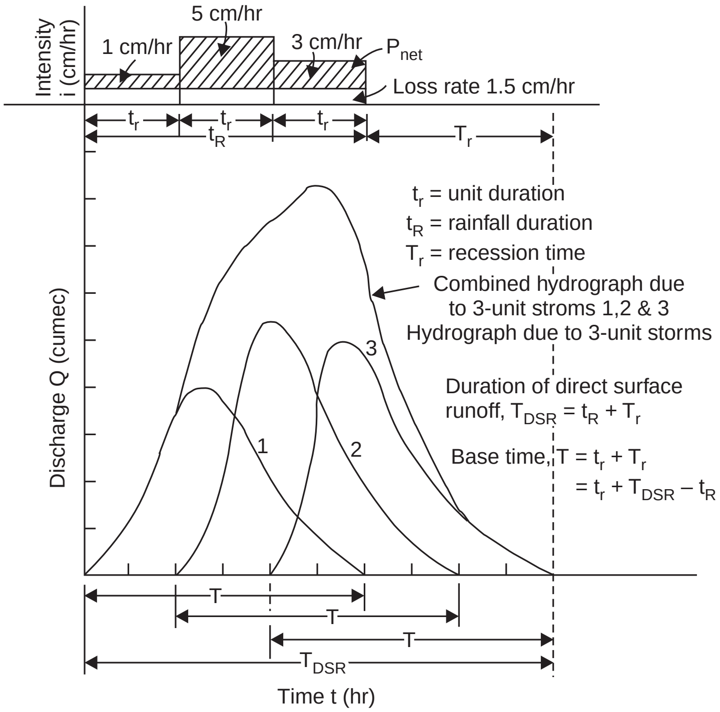

Fig. 5.24 Combined hydrograph (Example 5.7)

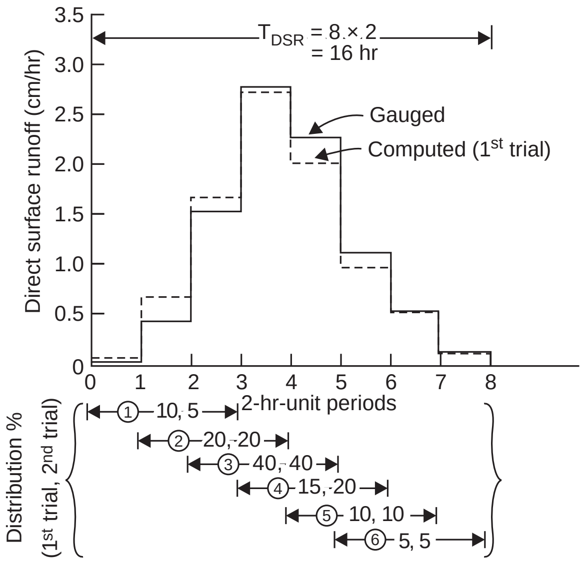

Fig. 5.25 Derivation of distribution percentages (Example 5.7)

Table 5.8 Derivation of distribution percentages by trials (Example 5.7)

		2-hr-unit periods								
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	
Rainfall rate (cm/hr) →		2.5	6.5	4.5						
Loss rate (cm/hr) →		1.5	1.5	1.5						
Net rain (cm/hr) →		1	5	3 (= 9 × 2 cm)						
Unit distribution				Distribution (cm/hr)						
Periods	Percentage									
1	10 (5)	(0.05)	(0.25)	(0.15)						
		0.10	0.50	0.30						
2	20		0.20	1.00	0.60					
3	40			0.40	2.00	1.20				
4	15 (20)				(0.20)	(1.00)	(0.60)			
					0.15	0.75	0.45			
5	10					0.10	0.50	0.30		
6	5						0.05	0.25	0.15	
Total	100 (100) cm/hr	0.10	0.70	1.70	2.75	2.05	1.00	0.55	0.15	(= 9 × 2 cm check)
		(0.05)	(0.45)	(1.55)	(2.80)	(2.30)	(1.15)			
	cumec	446	4015	1382	25000	20520	10260	4900	1338	(446/8920 = 0.05 cm/hr)

Note Figures in brackets indicate the adjusted values in the second trial.

and the direct surface runoff for successive 2-hr periods are 0.05, 0.45, 1.55, 2.80, 2.30, 1.15, 0.55, and 0.15 cm/hr. The first trial hydrograph computed in Table 5.8 is shown by dashed lines in Fig. 5.25 for comparison and selection of the distribution percentages for the second trial. The first percentage affects the first 3 unit periods, the second percentage affects the 2nd, 3rd and 4th unit periods and like that. Since the first trial hydrograph gives higher values (than gauged) for the first three unit periods, a lower percentage of 5 (instead of 10%) is tried. Similarly, the other percentages are adjusted till the computed discharge values agree with the gauged values. Thus, the second trial distribution percentages are 5, 20, 40, 20, 10, 5 which total 100 and are final and the distribution graph thus derived is shown in Fig. 5.26. In most cases, more trials are required to obtain the desired degree of accuracy.

Also see Appendix—G.

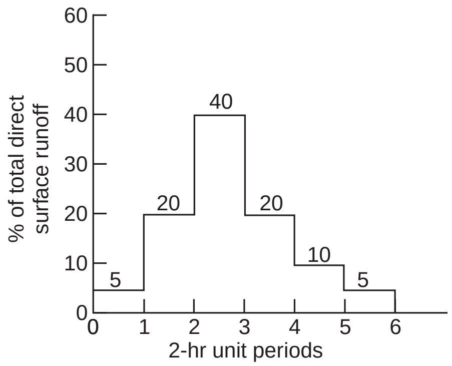

Fig. 5.26 Derived distribution graph (Example 5.7)

5.8 INSTANTANEOUS UNIT HYDROGRAPH

The difficulty of using a unit hydrograph of a known duration has been obviated by the development of the instantaneous unit hydrograph (IUH). The IUH is a hydrograph of runoff resulting from the instantaneous application of 1 cm net rain on the drainage basin. The IUH in conjunction with the design storm can be used to obtain the design flood by using a convolution integral. The IUH was first proposed by Clark in 1945. The IUH can be developed either directly from the observed data or by adopting conceptual models [see Chapter–16 & 17 for Methods of Determining IUH].

5.9 SYNTHETIC UNIT HYDROGRAPHS

In India, only a small number of streams are gauged (*i.e.,* streamflows due to single and multiple storms, are measured). There are many drainage basins (catchments) for which no streamflow records are available and unit hydrographs may be required for such basins. In such cases, hydrographs may be synthesised directly from other catchments, which are hydrologically and meteorologically homogeneous, or indirectly from other catchments through the application of empirical relationship. Methods for synthesising hydrographs for ungauged areas have been developed from time to time by Bernard, Clark, McCarthy and Snyder. The best known approach is due to Snyder (1938). Snyder analysed a large number of hydrographs from drainage basins in the Applachian Mountain region in USA ranging in the area from 25 to 25000 km^2 and selected the three parameters for the development of unit hydrograph, namely, base width (T), peak discharge (Q_p) and lag time (basin lag, t_p), Fig. 5.27, and proposed the following empirical formulae for the three parameters:

Lag time, $$t_p = C_t\,(L\,L_{ca})^{0.3} \qquad ...(5.8)$$

standard duration of net rain, $$t_r = \frac{t_p}{5.5} \qquad ...(5.9)$$

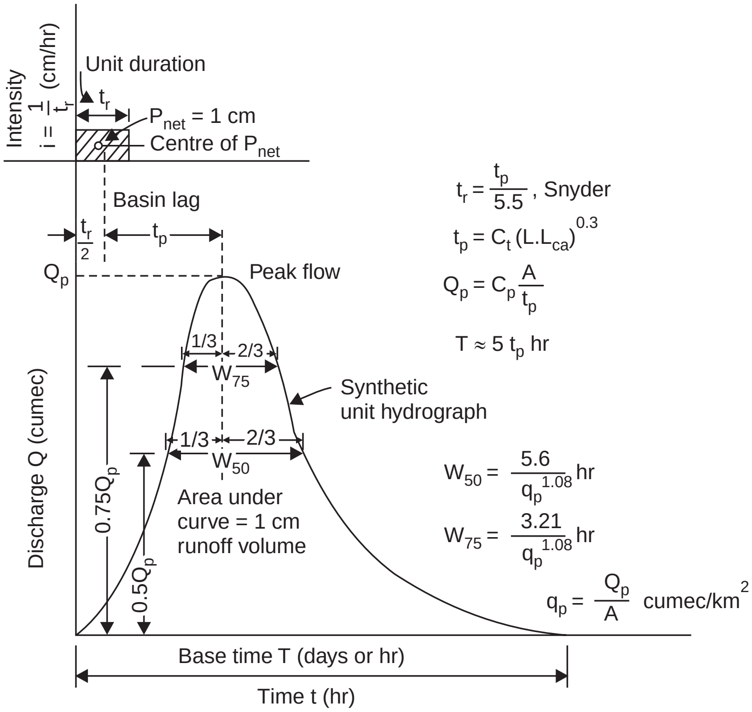

Fig. 5.27 Synthetic unit hydrograph parameters

For this standard duration of net rain,

peak flow, $$Q_p = C_p\,\frac{A}{t_p} \qquad ...(5.10)$$

time base in days, $$T = 3 + 3\left(\frac{t_p}{24}\right) \qquad ...(5.11)$$

peak flow per km² of basin, $$q_p = \frac{Q_p}{t_p} \qquad ...(5.12)$$

Snyder proposed subsequently an expression to allow for some variation in the basin lag with variation in the net rain duration, *i.e.,* if the actual duration of the storm is not equal to t_r given by Eq. (5.9) but is t_r', then

$$t_{pr} = t_p + \frac{t_r' - t_r}{4} \qquad ...(5.13)$$

where t_{pr} = basin lag for a storm duration of t_r', and t_{pr} is used instead of t_p in Eqs. (5.10), (5.11) and (5.12).

In the above equations,

t_p = lag time (basin lag), hr

C_t, C_p = empirical constants ($C_t \approx 0.2$ to 2.2, $C_p \approx 2$ to 6.5, the values depending on the basin characteristics and units)

A = area of the catchment (km^2)

L = length of the longest water course, *i.e.,* of the mainstream from the gauging station (outlet or measuring point) to its upstream boundary limit of the basin, (km) (Fig. 5.28)

L_{ca} = length along the main stream from the gauging station (outlet) to a point on the stream opposite the areal centre of gravity (centroid) of the basin

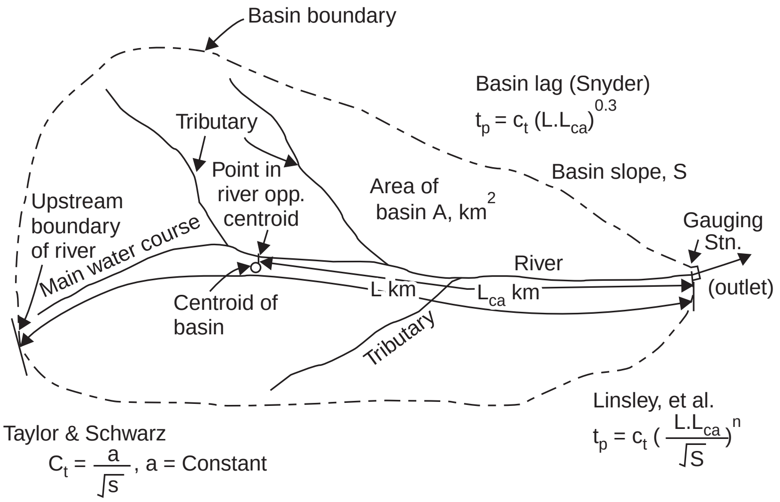

Fig. 5.28 Basin characteristics (Snyder)

Snyder considered that the shape of the unit hydrograph is likely to be affected by the basin characteristics like area, topography, shape of the slope, drainage density and channel storage. He dealt with the size and shape of basin by measuring the length of the mainstream channel. The coefficient C_t reflects the size, shape and slope of the basin.

Linsley, Kohler and Paulhus gave an expression for the lag time in terms of the basin characteristics (see Fig. 5.31) as

$$t_p = C_t \left(\frac{LL_{ca}}{\sqrt{S}}\right)^n \qquad \text{...(5.14)}$$

where S = basin slope, and the values of n and C_t, when L, L_{ca} were measured in miles are

$n = 0.38$

$C_t = 1.2$, for mountainous region

$= 0.72$, for foot hill areas

$= 0.35$, for valley areas

Taylor and Scwarz found from an analysis of 20 drainage basins of size 50-4000 km^2 in the north and middle Atlantic States in USA that (when L and L_{ca} were measured in miles)

$$C_t = \frac{0.6}{\sqrt{S}} \qquad \text{...(5.15)}$$

Time base in hr, $$T = 5\left(t_{pr} + \frac{t'_r}{2}\right) \quad ...(5.16)$$

i.e., $T = 5 \times t_{\text{peak}}$ ...(5.16 *a*)

The usual procedure for developing a synthetic unit hydrograph for a basin for which the streamflow records are not available is to collect the data for the basin like A, L, L_{ca} and to get the coefficients, C_t and C_p from adjacent basins whose streams are gauged and which are hydrometeorologically homogeneous. From these the three parameters, *i.e.,* the time to peak, the peakflow and the time base are determined from the Snyder's empirical equations, and the unit hydrograph can be sketched so that the area under the curve is equal to a runoff volume of 1 cm. Empirical formulae have been developed by the US Army Corps of Engineers (1959) for the widths of W_{50} and W_{75} of the hydrograph in hours at 50% and 75% height of the peak flow ordinate, respectively, (see Fig. 5.29) as

$$W_{50} = \frac{5.6}{q_p^{1.08}} \quad ...(5.17)$$

$$W_{75} = \frac{3.21}{q_p^{1.08}} = \frac{W_{50}}{1.75} \quad ...(5.18)$$

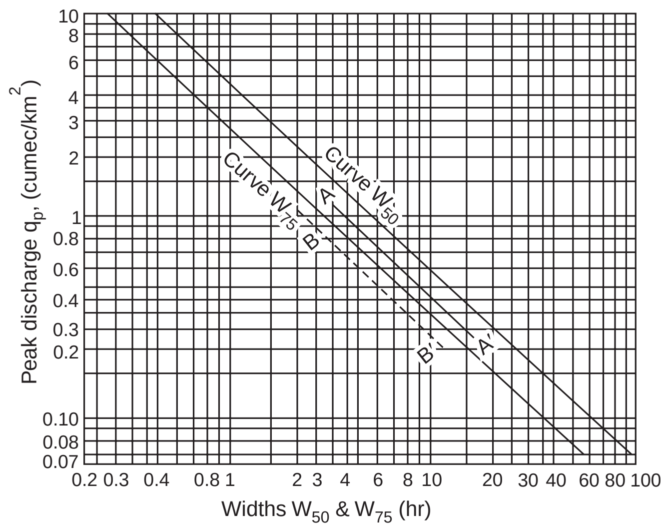

Fig. 5.29 Widths W_{50} and W_{75} for synthetic UG (US Army, 1959)

A still better shape of the unit hydrograph can be sketched with these widths (Fig. 5.27). The base time T given by Eq. (5.11) gives a minimum of 3 days even for very small basins and is in much excess of delay attributable to channel storage. In such cases, the author feels T given by Eq. (5.16 *a*) may be adopted and the unit hydrograph sketched such that the area under the curve gives a runoff volume of 1 cm.

Synthetic unit hydrographs for a few basins in India have been developed by CWPC.

Example 5.8 *The following are the ordinates of the 9-hour unit hydrograph for the entire catchment of the river Damodar up to Tenughat dam site:*

Time (hr):	*0*	*9*	*18*	*27*	*36*	*45*	*54*	*63*	*72*	*81*	*90*
Discharge (cumec):	*0*	*69*	*1000*	*210*	*118*	*74*	*46*	*26*	*13*	*4*	*0*

and the catchment characteristics are

$$A = 4480\ km^2,\ L = 318\ km,\ L_{ca} = 198\ km$$

Derive a 3-hour unit hydrograph for the catchment area of river Damodar up to the head of Tenughat reservoir, given the catchment characteristics as:

$$A = 3780\ km^2,\ L = 284\ km,\ L_{ca} = 184\ km$$

Use Snyder's approach with necessary modifications for the shape of the hydrograph.

Solution The 9-hr UG is plotted in Fig. 5.30 and from that t_p = 13.5 hr

t_r = 9 hr, $\quad \dfrac{t_p}{5.5} = \dfrac{13.5}{5.5} = 2.46$ hr $\neq t_r$ of 9 hr

$\therefore \quad t_r'$ = 9 hr, t_{pr} = 13.5 hr and t_p has to be determined

$$t_{pr} = t_p + \frac{t_r' - t_r}{4}$$

$$13.5 = t_p + \frac{9 - t_p/5.5}{4}$$

$\therefore \quad t_p$ = 11.8 hr

$$t_p = C_t\,(LL_{ca})^{0.3}$$

$$11.8 = C_t\,(318 \times 198)^{0.3}$$

$\therefore \quad C_t = 0.43$

Peak flow, $\quad Q_p = C_p \dfrac{A}{t_{pr}}$

$$1000 = C_p \frac{4480}{13.5}$$

$\therefore \quad C_p$ = 3.01, say, 3

The constants of $C_t = 0.43$ and $C_p = 3$ can now be applied for the catchment area up to the head of the Tenughat reservoir, which is meteorologically and hydrologically similar.

$$t_p = C_t\,(LL_{ca})^{0.3} = 0.43\,(284 \times 184)^{0.3} = 11.24 \text{ hr}$$

$\dfrac{t_p}{5.5} = \dfrac{11.24}{5.5} = 2.04$ hr $\neq t_r$ of 3 hr (duration of the required UG)

$\therefore \quad t_r'$ = 3 hr, t_r = 2.04 hr and t_{pr} has to be determined.

$$t_{pr} = t_p + \frac{t_r' - t_r}{4} = 11.24 + \frac{3 - 2.04}{4}$$

$\therefore \quad t_{pr}$ = 11.48 hr, say, **11.5 hr**

Peak flow $Q_p = C_p \dfrac{A}{t_{pr}} = 3 \times \dfrac{3780}{11.5} =$ **987 cumec**

Time to peak from the beginning of rising limb

$$t_{\text{peak}} = t_{pr} + \frac{t_r'}{2} = 11.5 + \frac{3}{2} = \mathbf{13\ hr}$$

Time base (Snyder's) T (days) $= 3 + 3\left(\dfrac{t_{pr}}{24}\right) = 3 + 3\left(\dfrac{11.5}{24}\right) = 4.44$ days or 106.5 hr

This is too long a runoff duration and hence to be modified as

$$T(\text{hr}) = 5 \times t_{\text{peak}} = 5 \times 13 = \mathbf{65\ hr}$$

To obtain the widths of the 3-hr UG at 50% and 75% of the peak ordinate :

$$q_p = \frac{Q_p}{A} = \frac{987}{3780} = 0.261 \text{ cumec/km}^2$$

$$W_{50} = \frac{5.6}{(q_p)^{1.08}} = \frac{5.6}{(0.261)^{1.08}} = \mathbf{23.8\ hr}$$

$$W_{75} = \frac{3.21}{(q_p)^{1.08}} = \frac{3.21}{(0.261)^{1.08}} = \mathbf{13.6\ hr} = \frac{23.8}{1.75}$$

These widths also seem to be too long and a 3-hr UG can now be sketched using the parameters Q_p = 987 cumec, t_{peak} = 13 hr and T = 65 hr such that the area under the UG is equal to a runoff volume of 1 cm, as shown in Fig. 5.30.

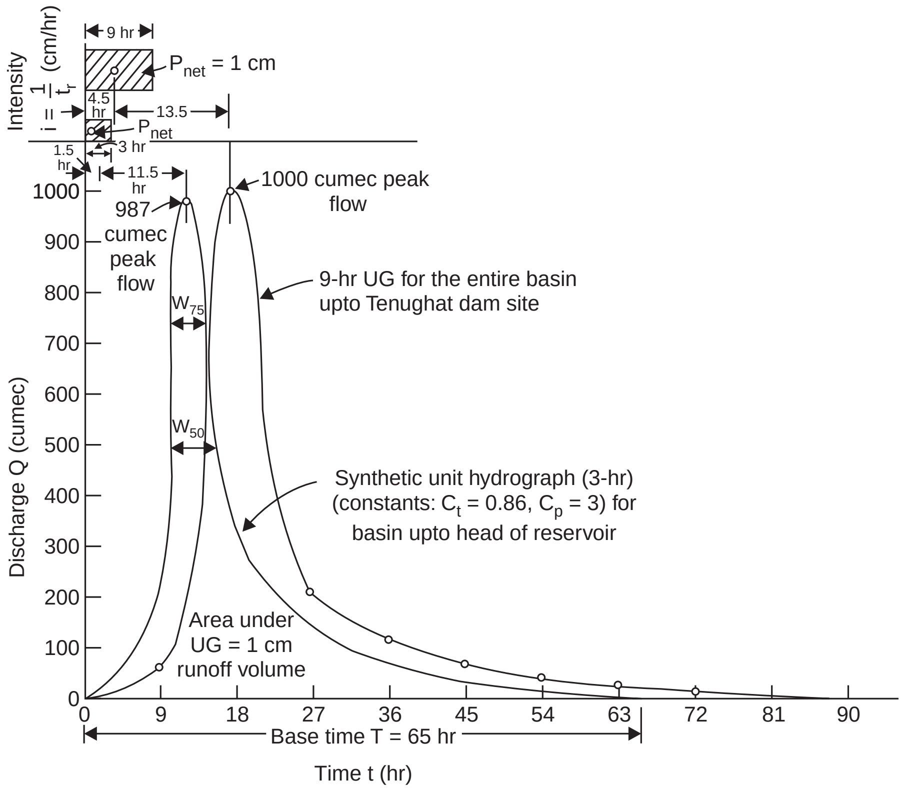

Fig. 5.30 Synthetic unit hydrograph derived (Example 5.8)

5.10 TRANSPOSING UNIT HYDROGRAPHS

From Eq. (5.14)

$$t_p = C_t \left(\frac{LL_{ca}}{\sqrt{S}}\right)^n$$

$$\underbrace{\log t_p}_{y} = \underbrace{\log C_t}_{c} + \underset{\downarrow}{n}\,\underset{m}{} \log \underbrace{\frac{LL_{ca}}{\sqrt{S}}}_{x}$$

$$y \;=\; c + \;\; m \;\; x$$

Hence, a plot of t_p vs. $\frac{LL_{ca}}{\sqrt{S}}$ on log-log paper from data from basins of similar hydrologic characteristics gives a straight-line relationship (Fig. 5.31). The constant $C_t = t_p$ when $\frac{LL_{ca}}{\sqrt{S}}$ = 1, and the slope of the straight line gives n. It may be observed that for basins having different hydrologic characteristics the straight lines obtained are nearly parallel, *i.e.,* the values of C_t varies depending upon the slope of the basin (as can be seen from Eq. 5.15) but the value of n is almost same.

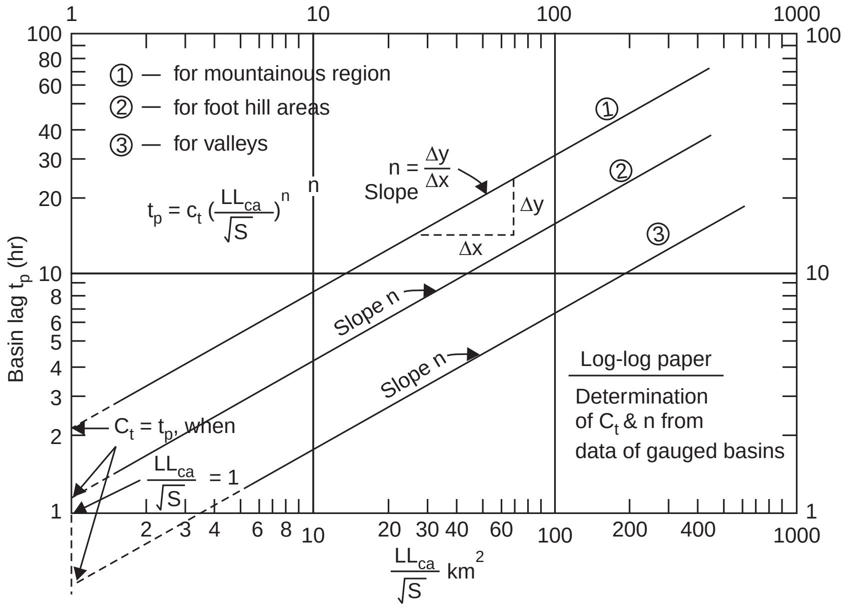

Fig. 5.31 Basin lag vs. basin characteristics

From a plot of q_p vs. t_p, or from dimensionless hydrographs from gauged basins, the peak flow and the shape of the unit hydrograph for ungauged basins can be estimated provided they are hydrometeorologically the same. The dimensionless unit hydrographs eliminates the effect of basin size and much of the effect of basin shape. Hence, it is a useful means for comparison of unit hydrographs of basins of different sizes and shapes or those resulting from different storm patterns. It can be derived from a unit hydrograph by reducing its time and discharge scales by dividing by t_p and Q_p, respectively. By averaging a number of dimensionless unit hydrographs of drainage areas, a representative dimensionless graph can be synthesized for a particular hydrological basin. A unit hydrograph for an ungauged basin, hydrologically similar, can be obtained directly from this dimensionless graph by multiplying by the appropriate values of t_p (obtained from log-log plot) and Q_p (from a plot of q_p vs. t_p).

Example 5.9 *For the 9-hr UG given for the entire catchment of the river Damodar in Example 5.8, derive a dimensionless unit hydrograph.*

Solution From example 5.8, t_p = 11.8 hr, Q_p = 1000 cumec, and hence the following computation can be made:

$\frac{t}{t_p}$:	0	0.763	1.525	2.29	3.05	3.82	4.58	5.34	6.10	6.87	7.63
$\frac{Q}{Q_p}$:	0	0.069	1	0.21	0.118	0.074	0.046	0.026	0.013	0.004	0

The dimensionless unit hydrograph is plotted as shown in Fig. 5.32.

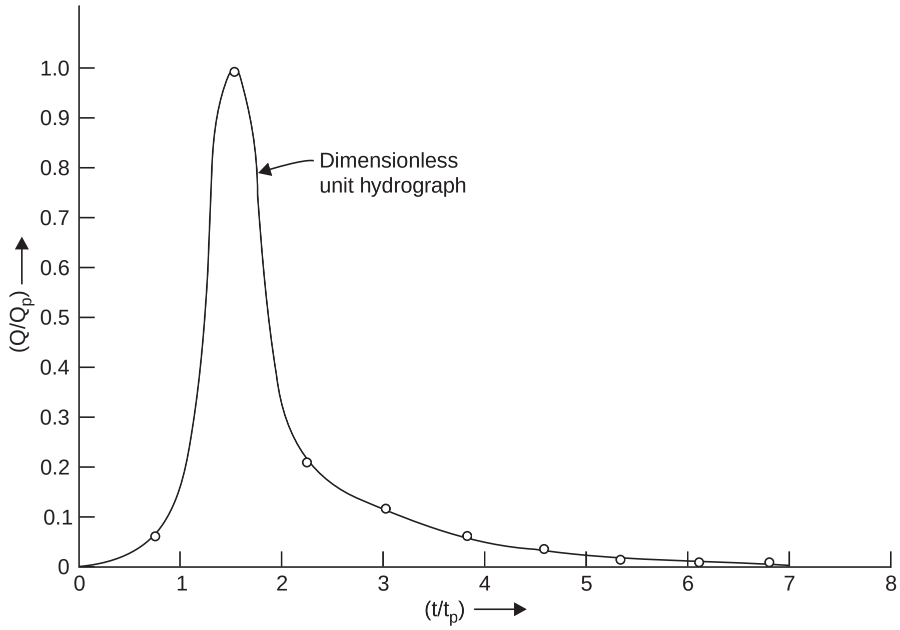

Fig. 5.32 Dimensionless unit hydrograph

The US Soil Conservation Service (1971), using many hydrographs from drainage areas of varying sizes and different geographical locations, has developed a dimensionless unit hydrograph and has developed the following formulae.

$$t_{\text{peak}} = \frac{t_c + 0.133\, t_c}{1.7} \quad \text{and} \quad q_p = \frac{0.208\, AQ}{t_{\text{peak}}}$$

where t_c = time of concentration (hr)

t_{peak} = time to peak discharge from beginning (hr)

$= \frac{t_r}{2} + t_p$, where t_p is lag time

q_p = peak discharge (cumec)

A = drainage area (km^2)

Q = quantity of runoff, which is 1 mm for the unit hydrograph

Most hydrologists assume t_c as the time from the end of rainfall excess (P_{net}) to the point of inflection on the falling limb of the hydrograph; t_{peak} can be determined from t_c or lag time t_p. (basin lag) for the required t_r-hr *UG*. From t_{peak} and known drainage area q_p can be determined. Once t_{peak} and q_p are obtained, 't vs. q' can be computed from the data of the dimensionless unit hydrograph and the *UG* can be sketched.

Example 5.9 (a) *Construct a 4-hr UH for a drainage basin of 200 km² and lag time 10 hr by the SCS method, given (pk = peak):*

$\frac{t}{t_{pk}}$:	0.5	1	2	3	4	5
$\frac{Q}{Q_p}$:	0.4	1	0.32	0.075	0.018	0.004

Solution $$t_{pk} = \frac{t_r}{2} + t_p = \frac{4}{2} + 10 = 12 \text{ hr}$$

(*i*) $Q_p = \dfrac{5.36\,A}{t_{pk}} = \dfrac{5.36 \times 200}{12} = \mathbf{89.33}$ cumec, which occurs at $\dfrac{t}{t_{pk}} = 1$

or $t = t_{pk} = \mathbf{12\ hr}$

(*ii*) At $\dfrac{t}{t_{pk}} = 0.5$ or $t = 0.5 \times 12 = \mathbf{6\ hr,}$ $\dfrac{Q}{Q_p} = 0.4$ or $Q = 0.4 \times 89.33 = \mathbf{35.732}$ cumec

(*iii*) At $\dfrac{t}{t_{pk}} = 2$ or $t = 2 \times 12 = \mathbf{24\ hr,}$ $\dfrac{Q}{Q_p} = 0.32$ or $Q = 0.32 \times 89.33 = \mathbf{28.6}$ cumec

(*iv*) At $\dfrac{t}{t_{pk}} = 3$ or $t = 3 \times 12 = \mathbf{36\ hr,}$ $\dfrac{Q}{Q_p} = 0.075$, or $Q = 0.075 \times 89.33 = \mathbf{6.7}$ cumec

Time base $T = 5\,t_{pk} = 5 \times 12 = 60$ hr; $W_{75} = W_{50}/1.75$

With this, a 4-hr UH can be sketched.

5.11 APPLICATION OF UNIT HYDROGRAPH

The application of unit hydrograph consists of two aspects:

(*i*) From a unit hydrograph of a known duration to obtain a unit hydrograph of the desired duration, either by the *S*-curve method or by the principle of superposition.

(*ii*) From the unit hydrograph so derived, to obtain the flood hydrograph corresponding to a single storm or multiple storms. For design purposes, a design storm is assumed, which with the help of unit hydrograph, gives a design flood hydrograph.

While the first aspect is already given, the second aspect is illustrated in the following example.

Example 5.10 *The 3-hr unit hydrograph ordinates for a basin are given below. There was a storm, which commenced on July 15 at 16.00 hr and continued up to 22.00 hr, which was followed by another storm on July 16 at 4.00 hr which lasted up to 7.00 hr. It was noted from the mass curves of self-recording raingauge that the amount of rainfall on July 15 was 5.75 cm from 16.00 to 19.00 hr and 3.75 cm from 19.00 to 22.00 hr, and on July 16, 4.45 cm from 4.00 to 7.00 hr. Assuming an average loss of 0.25 cm/hr and 0.15 cm/hr for the two storms, respectively,* *and a constant base flow of 10 cumec, determine the stream flow hydrograph and state the time of occurrence of peak flood.*

Time (hr):	*0*	*3*	*6*	*9*	*12*	*15*	*18*	*21*	*24*	*27*
UGO (cumec):	*0*	*1.5*	*4.5*	*8.6*	*12.0*	*9.4*	*4.6*	*2.3*	*0.8*	*0*

Solution Since the duration of the UG is 3 hr, the 6-hr storm (16.00 to 22.00 hr) can be considered as 2-unit storm producing a net rain of 5.75 – 0.25 × 3 = 5 cm in the first 3-hr period and a net rain of 3.75 – 0.25 × 3 = 3 cm in the next 3-hr period. The unit hydrograph ordinates are multiplied by the net rain of each period lagged by 3 hr. Similarly, another unit storm lagged by 12 hr (4.00 to 7.00 hr next day) produces a net rain of 4.45 – 0.15 × 3 = 4 cm which is multiplied by the *UGO* and written in col (5) (lagged by 12 hr from the beginning), Table 5.9. The rainfall excesses due to the three storms are added up to get the total direct surface discharge ordinates. To this, the base flow ordinates (BFO = 10 cumec, constant) are added to get the total discharge ordinates (stream flow).

The flood hydrograph due to the 3 unit storms on the basin is obtained by plotting col (8) vs. col. (1) (Fig. 5.33). This example illustrates the utility of the unit hydrograph in deriving flood hydrographs due to a single storm or multiple storms occurring on the basin.

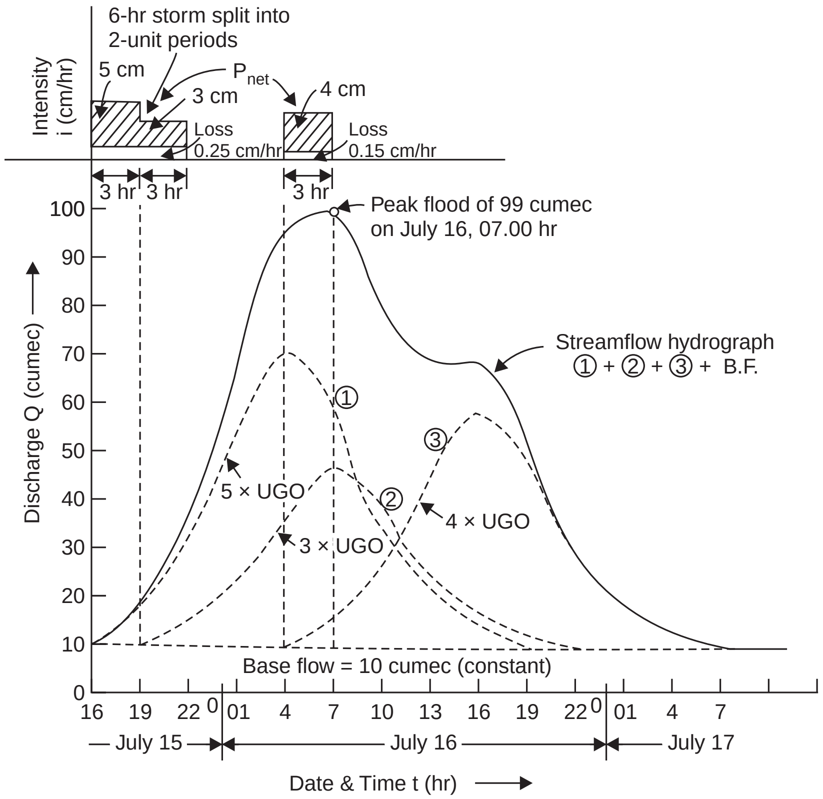

Fig. 5.33 Application of UG to obtain stream flow hydrograph (Example 5.10)

Table 5.9 Derivation of the flood hydrograph due to multiple storms (Example 5.10)

Time	*UGO**	*DRO due to rainfall excess*			*Total*			
		I	*II*	*III*	*DRO*	*BFO*	*TRO*	*Remarks*
(hr)		*UGO × 5 cm*	*UGO × 3 cm*	*UGO × 4 cm*	*(3) + (4) + (5)*	*(constant)*	*(6) + (7)*	
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	
0	0	0	—	—	0	10	10.0	– July 15, 16.00 hr
3	1.5	7.5	0	—	7.5	10	17.5	commencement of flood
6	4.5	22.5	4.5	—	27.0	10	37.0	
9	8.6	43.0	13.5	—	56.5	10	66.5	
12	12.0	60.0	25.8	0	85.8	10	95.8	
15	9.4	47.0	36.0	6	89.0	10	99.0	— Peak flood on July 16,
18	4.6	23.0	28.2	18	69.2	10	79.2	07.00 hr
21	2.3	11.5	13.8	34.4	59.7	10	69.7	
24	0.8	4.0	6.9	48	58.9	10	68.9	
27	0	0	2.4	37.6	40.0	10	50.0	
30			0	18.4	18.4	10	28.4	
33				9.2	9.2	10	19.2	
36				3.2	3.2	10	13.2	
39				0	0	10	10.0	— Flood subsides on July 17
								07.00 hr

*All columns are in cumec units except col (1).

Example 5.11 *The design storm of a water shed has the depths of rainfall of 4.9 and 3.9 cm for the consecutive 1-hr periods. The 1-hr UG can be approximated by a triangle of base 6 hr with a peak of 50 cumec occurring after 2 hr from the beginning. Compute the flood hydrograph assuming an average loss rate of 9 mm/hr and constant base flow of 10 cumec. What is the area of water shed and its coefficient of runoff?*

Solution (*i*) The flood hydrograph due to the two consecutive hourly storms is computed in Table 5.10, Fig. P5.34.

(*ii*) Area of water shed—To produce 1-cm net rain over the entire water shed (A km^2).

Volume of water over basin = Area of UG (triangle)

$$(A \times 10^6)\,\frac{1}{100} = \tfrac{1}{2}\,(6 \times 60 \times 60)\,50$$

from which, A = **54 km²**

(*iii*) Coefficient of runoff $$C = \frac{R}{P} = \frac{(4.9 - 0.9) + (3.9 - 0.9)}{4.9 + 3.9} = \mathbf{0.795}$$

Table 5.10 Computation of design flood hydrograph (Example 5.11)

Time (hr)	*UGO* (cumec)*	*DRO due to rainfall excess (cumec)* 4.9 – 0.9 = 4 cm	*DRO due to rainfall excess (cumec)* 3.9 – 0.9 = 3 cm	*Total (cumec)*	*BF (cumec)*	*TRO (cumec)*	*Remarks*
1	0	0	—	0	10	10	
2	25	100	0	100	10	110	
3	50	200	75	275	10	285	
4	37.5	150	150	300	10	**310**	← Peak flood†
5	25	100	112.5	212.5	10	222.5	
6	12.5	50	75	125	10	135	
7	0	0	37.5	37.5	10	47.5	
8	—	—	0	0	10	10	

*Ordinates by proportion in the triangular UG. †–Peak flood of 310 cumec, after 4 hr from the commencement of the storm.

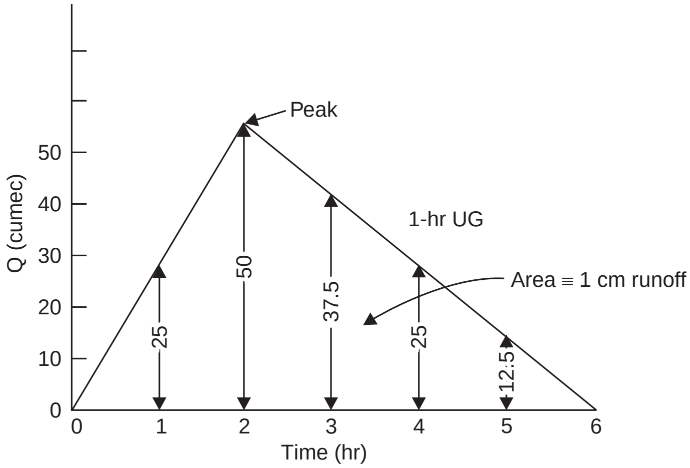

Fig. 5.34 (Example 5.11)

Table 5.11 Computation of stream flow from distribution percentages. (Example 5.12)

Time (hr)	*Distribution percentages*	*Rainfall excess (cm)*	*DRO due to rainfall excess (cm)*			*Total DRO*		*BF (cumec)*	*Stream flow (cumec)*	*Remarks*
		P-loss = P_{net}	*2*	*7*	*4*	*(cm)*	*(cumec)*			
1	5	3.2 – 1.2 = 2	0.10	—	—	0.10	12.5*	10	22.5 ←	June 4, 1982 04.00 hr
2	20	8.2 – 1.2 = 7	0.40	0.35	—	0.75	93.75	10	103.75	Commencement of flood
3	40	5.3 – 1.2 = 4	0.80	1.40	0.20	2.40	300	10	310	
4	20		0.40	2.80	0.80	4.00	500	10	510 ←	Peak flood at 07.00 hr on June 4, 1982
5	10		0.20	1.40	1.60	3.20	400	10	410	
6	5		0.10	0.70	0.80	1.60	200	10	210	
7	—		—	0.35	0.40	0.75	93.75	10	103.75	
8	—		—	—	0.20	0.20	25	10	35	
Total	100	13	2.00	7.00	4.00	13.00	1625			

$* \ \frac{0.10}{100} \frac{(45 \times 10^6)}{1 \times 60 \times 60} = 12.5$ cumec.

Example 5.12 *Storm rainfalls of 3.2, 8.2 and 5.2 cm occur during three successive hours over an area of 45 km^2. The storm loss rate is 1.2 cm/hr. The distribution percentages of successive hours are 5, 20, 40, 20, 10 and 5. Determine the streamflows for successive hours assuming a constant base flow of 10 cumec. State the peak flow and when it is expected; the precipitation started at 04.00 hr, on June 4, 1982.*

Solution The computation of stream flow hydrograph from the distribution percentages due to net rainfall in three successive hours (*i.e.,* from a complex storm) over an area of 45 km^2 is made in Table 5.11.

Example 5.13 *The successive three-hourly ordinates of a 6-hr UG for a particular basin are 0, 15, 36, 30, 17.5, 8.5, 3, 0 cumec, respectively. The flood peak observed due to a 6-hr storm was 150 cumec. Assuming a costant base flow of 6 cumec and an average storm loss of 6 mm/hr, determine the depth of storm rainfall and the streamflow at successive 3 hr interval.*

Solution DRO peak = Flood peak – BF

= 150 – 6 = 144 cumec

$$P_{\text{net}} = \frac{DRO_{\text{peak}}}{UG_{\text{peak}}} = \frac{144}{36} = 4 \text{ cm}$$

Depth of storm rainfall,

$$P = P_{\text{net}} + \text{losses} = 4 + 0.6 \times 6 = 7.6 \text{ cm.}$$

$$\text{DRO} = \text{UGO} \times P_{\text{net}}; \quad \text{DRO} + \text{BF} = \text{TRO}$$

Hence, multiplying the given UGO by 4 cm and adding 6 cumec, the stream flow ordinates at successive 3-hr intervals are: 6, 66, 150, 126, 76, 40, 18, 6 cumec, respectively.

QUIZ V

I Match the items in 'A' with items in 'B'

A	B
(*i*) Ground water depletion	(*a*) Unit storm
(*ii*) Unit hydrograph	(b) Flood hydrograph
(*iii*) TRO	(*c*) Bernard
(*iv*) UGO	(*d*) Synthetic unit hydrograph
(*v*) Unit duration	(*e*) S-curve hydrograph
(*vi*) Change of unit duration	(*f*) (TRO – BFO) ÷ P_{net}
(*vii*) Distribution percentages	(*g*) UGO × P_{net} + BFO
(*viii*) Design storm	(*h*) 1 cm of runoff
(*ix*) Ungauged stream	(*i*) Base flow recession curve

II Say 'true' or 'false', if false, give the correct statement.

(*i*) Unit hydrographs should be used for basins, larger than 5000 km^2.

(*ii*) Larger unit periods are required for larger basins.

(*iii*) If the peak of the unit hydrograph is 25 cumec, then the peak of the hydrograph producing 8 cm of runoff is 215 cumec, assuming a constant base flow of 15 cumec.