

Chapter 5

HYDROGRAPHS

5.1 HYDROGRAPH COMPONENTS

A *hydrograph* is a graph showing discharge (*i.e.*, stream flow at the concentration point) versus time. The various components of a natural hydrograph are shown in Fig. 5.1. At the beginning, there is only base flow (*i.e.*, the ground water contribution to the stream) gradually depleting in an exponential form. After the storm commences, the initial losses like interception and infiltration are met and then the surface flow begins. The hydrograph gradually rises

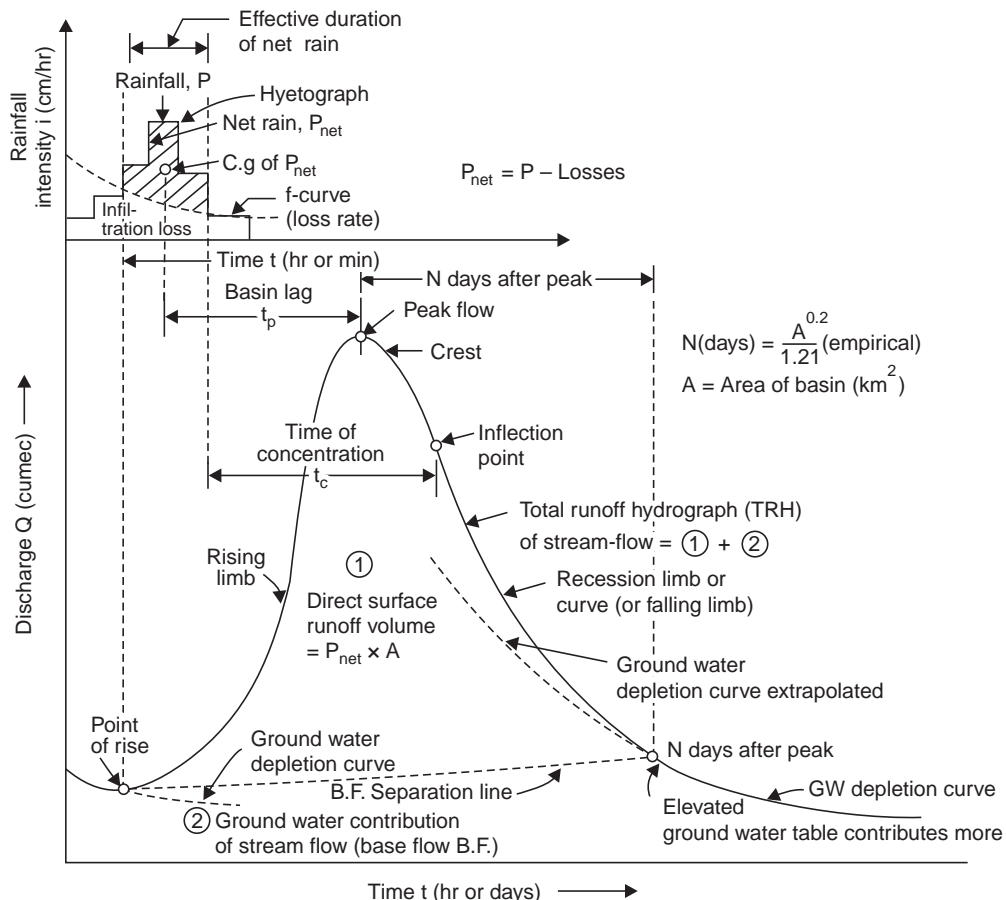


Fig. 5.1 Components of streamflow hydrograph

and reaches its peak value after a time t_p (called *lag time* or *basin lag*) measured from the centroid of the hyetograph of net rain. Thereafter it declines and there is a change of slope at the inflection point, *i.e.*, there has been, inflow of the rain up to this point and after this there is gradual withdrawal of catchment storage. By this time the ground water table has been built up by the infiltrating and percolating water, and now the ground water contributes more into the stream flow than at the beginning of storm, but thereafter the GWT declines and the hydrograph again goes on depleting in the exponential form called the *ground water depletion curve* or the *recession curve*. If a second storm occurs now, again the hydrograph starts rising till it reaches the new peak and then falls and the ground water recession begins, Fig. 5.2.

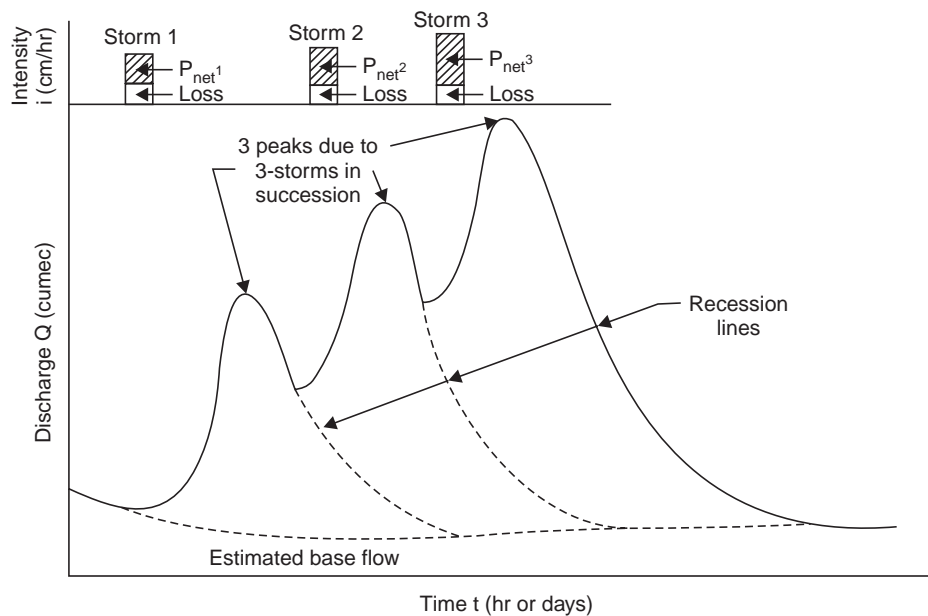


Fig. 5.2 Hydrograph with multiple peaks

Thus, in actual streams gauged, the hydrograph may have a single peak or multiple peaks according to the complexity of storms. For flood analysis and derivation of unit hydrograph, a single peaked hydrograph is preferred. A complex hydrograph, however, can be resolved into simple hydrographs by drawing hypothetical recession lines as shown in Fig. 5.2.

It has been found from many hydrographs that the ground water depletion curves for a given drainage basin are nearly the same and hence it is termed as the normal ground water depletion curve. It has been found that such curves, or at least their segments, follow a simple inverse exponential function of the elapsed time of the form.

$$Q_t = Q_0 K_r^{-t} \quad \dots(5.1)$$

where Q_0 = discharge at start of period

Q_t = discharge at end of time t

K_r = recession constant

As Q_t is the derivations of storage with respect to time,

$$-d S_t = Q_t dt$$

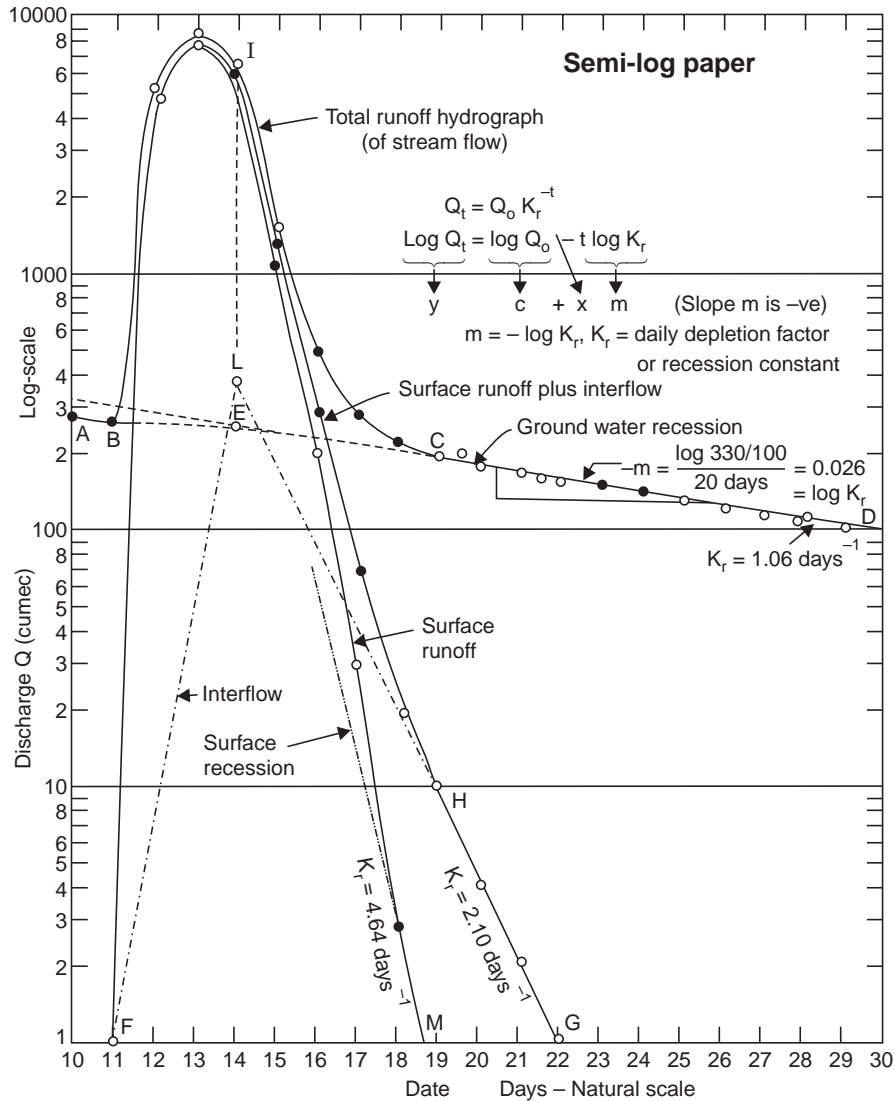


Fig. 5.5 Separation of streamflow components (after Barnes, 1940)

5.3 HYDROGRAPH SEPARATION

For the derivation of unit hydrograph, the base flow has to be separated from the total runoff hydrograph (*i.e.*, from the hydrograph of the gauged stream flow). Some of the well-known base flow separation procedures are given below, Fig. 5.6.

(i) Simply by drawing a line AC tangential to both the limbs at their lower portion. This method is very simple but is approximate and can be used only for preliminary estimates.

(ii) Extending the recession curve existing prior to the occurrence of the storm up to the point D directly under the peak of the hydrograph and then drawing a straight line DE, where E is a point on the hydrograph N days after the peak, and N (in days) is given by

$$N = 0.83 A^{0.2} \quad \dots(5.3)$$

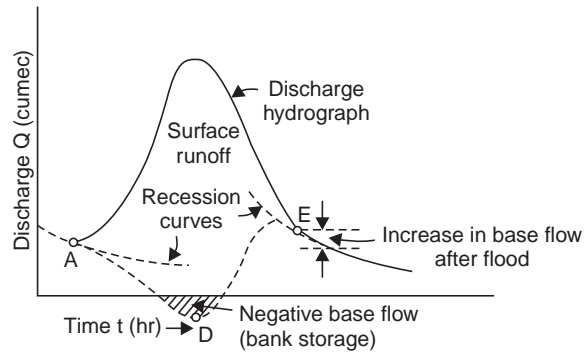


Fig. 5.7 Negative baseflow

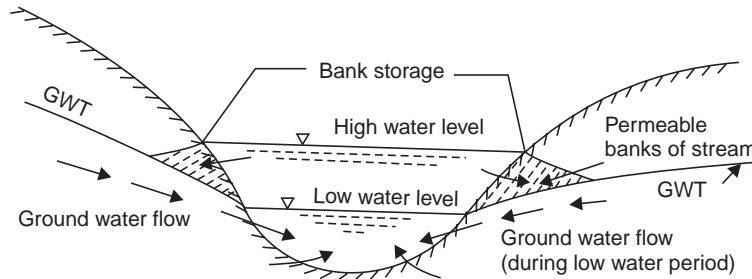


Fig. 5.8 Bank storage

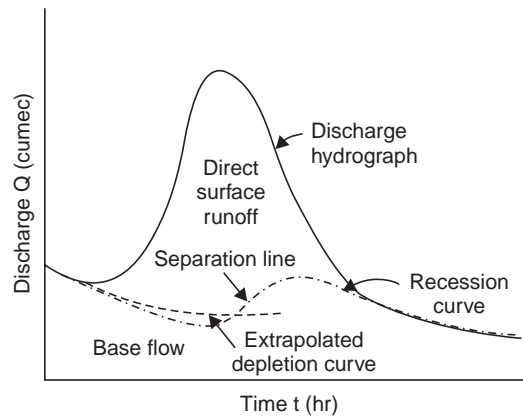


Fig. 5.9 Actual baseflow variation

5.4 UNIT HYDROGRAPH

The unit hydrograph is defined as the hydrograph of storm runoff resulting from an isolated rainfall of some unit duration occurring uniformly over the entire area of the catchment, produces a unit volume (*i.e.*, 1 cm) of runoff.

Derivation of the unit hydrographs. The following steps are adopted to derive a unit hydrograph from an observed flood hydrograph (Fig. 5.10).

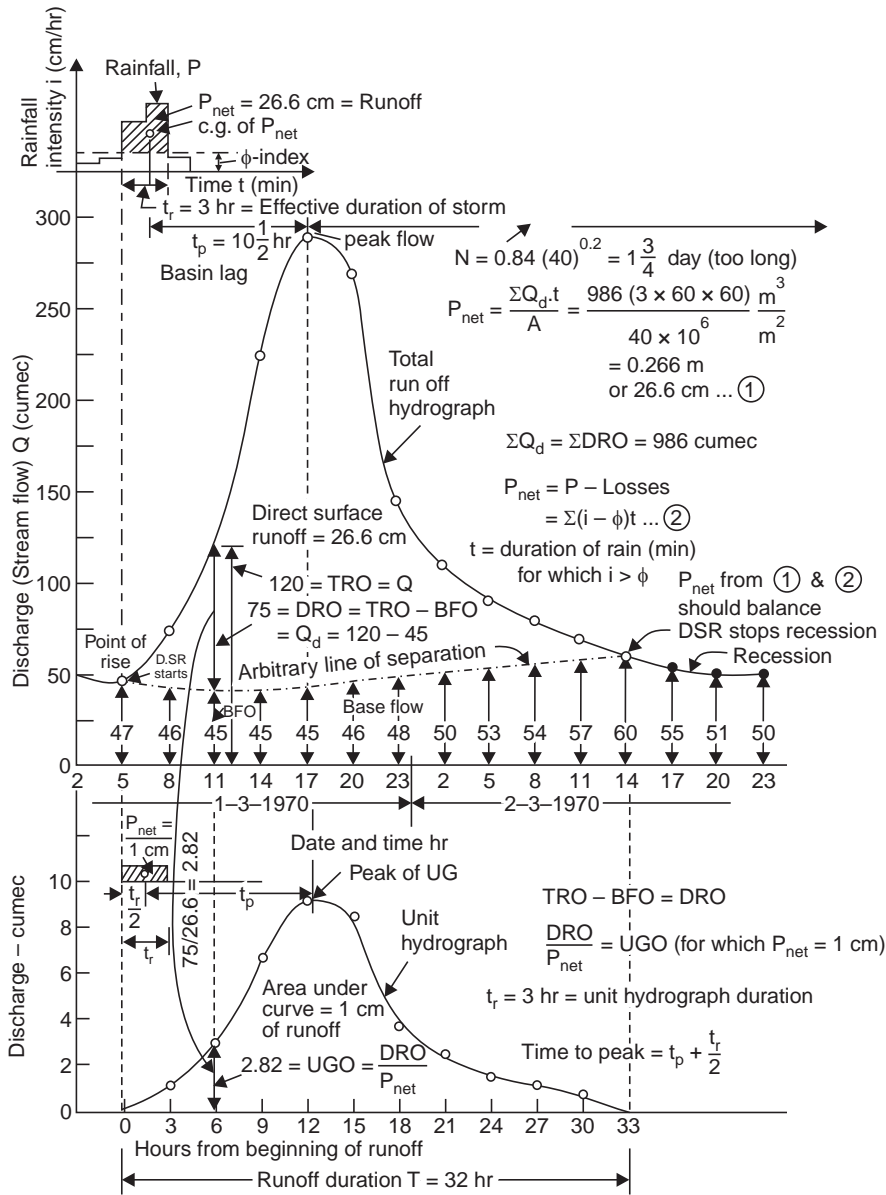


Fig. 5.10 Derivation of a unit hydrograph (Example 5.1)

(i) Select from the records isolated (single-peaked) intense storms, which occurring uniformly over the catchment have produced flood hydrographs with appreciable runoff (>1 cm, say, 8 to 16 cm). The unit period selected should be such that the excess rainfall (*i.e.*, P_{net}) occurs fairly uniformly over the entire drainage basin. Larger unit periods are required for larger basins. The unit periods may be in the range of 15-30% of the 'peak time' period, *i.e.*, the time from the beginning of surface runoff to the peak, and the typical unit periods may be 3, 6, 8, 12 hours. (The time of concentration may be a little longer than the peak time).

The unit storm is a storm of such duration that the period of surface runoff is not much less for any other storm of shorter duration.

(ii) Select a flood hydrograph, which has resulted from a unit storm chosen in item (i) above.

(iii) Separate the base flow from the total runoff (by the well-known base flow separation procedures).

(iv) From the ordinates of the total runoff hydrograph (at regular time intervals) deduct the corresponding ordinates of base flow, to obtain the ordinates of direct runoff.

(v) Divide the volume of direct runoff by the area of the drainage basin to obtain the net precipitation depth over the basin.

(vi) Divide each of the ordinates of direct runoff by the net precipitation depth to obtain the ordinates of the unit hydrograph.

(vii) Plot the ordinates of the unit hydrograph against time since the beginning of direct runoff. This will give the unit hydrograph for the basin, for the duration of the unit storm (producing the flood hydrograph) selected in item (i) above.

In unit hydrograph derivation, such storms should be selected for which reliable rainfall and runoff data are available. The net rain graph (hyetograph of excess rain) should be determined by deducting the storm loss and adjusting such that the total volume of net storm rain is equal to the total volume of direct surface runoff. The unit hydrograph derived, which, when applied to the known net rain data, should yield the known direct runoff hydrograph.

The steps given above for the derivation of unit hydrograph can be formulated as follows (exemplified in Fig. 5.10).

$$P_{\text{net}} = P - \text{Losses} \quad \dots(5.4)$$

$$\text{or } P_{\text{net}} = \frac{\sum Q_d t}{A}, \quad Q_d = DRO \quad \dots(5.4 a)$$

$$TRO - BFO = DRO \quad \dots(5.4 b)$$

$$\frac{DRO}{P_{\text{net}}} = UGO \quad \dots(5.4 c)$$

where P = total rainfall

P_{net} = net precipitation (from hyetograph) or direct runoff as equivalent depth over the basin.

Losses = due to infiltration (F_p), etc.

A = area of the drainage basin

Q_d , DRO = direct runoff ordinate

TRO = total runoff ordinate

t = time interval between successive direct runoff ordinates

BFO = base flow ordinate

Elements of unit hydrograph. The various elements of a unit hydrograph are shown in Fig. 5.11.

Base width (T)—The period of direct surface runoff (due to a unit storm) of the unit hydrograph is called the *time base* or the *base width*.

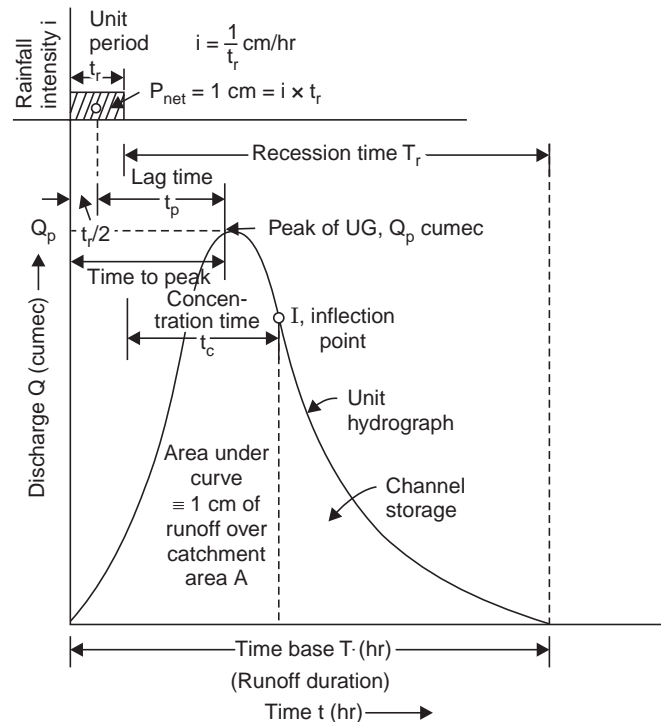


Fig. 5.11 Elements of unit hydrograph

Unit storm—The storm of unit duration (*i.e.*, duration of the unit hydrograph) regardless of its intensity is called *unit storm*.

Unit period—The time duration of the unit storm (*i.e.*, the duration of the unit hydrograph) is called *unit period*.

Lag time (t_p)—The time from the centre of a unit storm to the peak discharge of the corresponding unit hydrograph is called *lag time*.

Recession time (T_r)—The duration of the direct surface runoff after the end of the excess or net rainfall, is called *recession time in hydrograph analysis*.

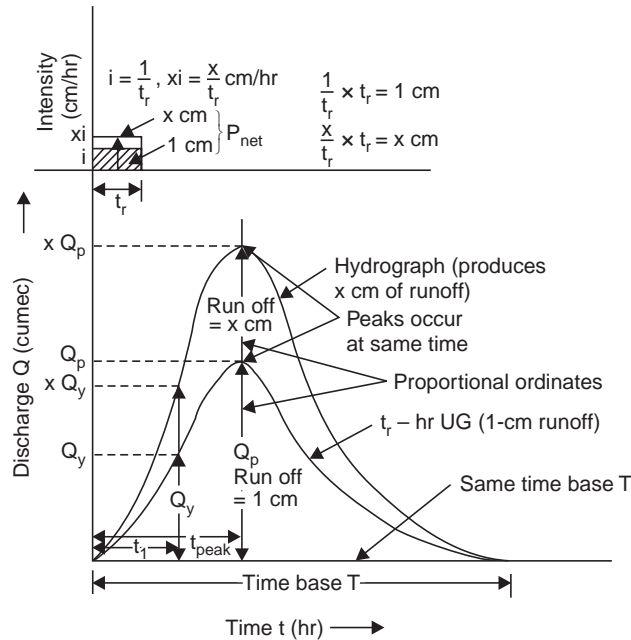
Propositions of the Unit Hydrograph

The following are the basic propositions of the unit hydrograph:

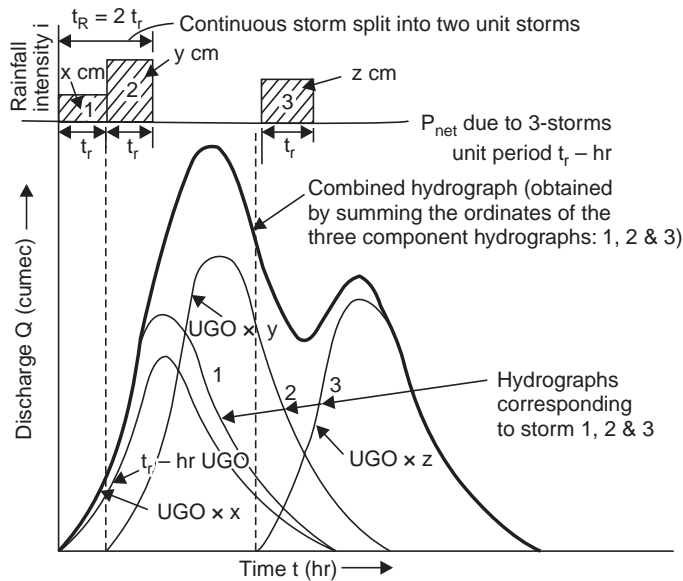
(i) *Same runoff duration*. For all unit storms of different intensities, the period of surface runoff (*i.e.*, time base, base width or base period) is approximately the same, although they produce different runoff volumes (Fig. 5.12 (a)).

(ii) *Proportional ordinates*. For unit storms of different intensities, the ordinates of the hydrograph at any given time, are in the same proportion as the rainfall intensities (Fig. 5.12 (a)).

(iii) *Principle of superposition*. If there is a continuous storm and/or isolated storms of uniform intensity net rain, they may be divided into unit storms and hydrographs of runoff for each storm obtained, and the ordinates added with the appropriate time lag to get the combined hydrograph (Fig. 5.12 (b)).



(a) Proportional ordinates and same time base



(b) Principle of superposition

Fig. 5.12 Propositions of unit hydrograph

(iv) *Same distribution percentages.* If the total period of surface runoff (*i.e.*, time base or base width) is divided into equal time intervals the percentage of surface runoff that occurs during each of these periods will be same for all unit storms of different intensities (Fig. 5.13).

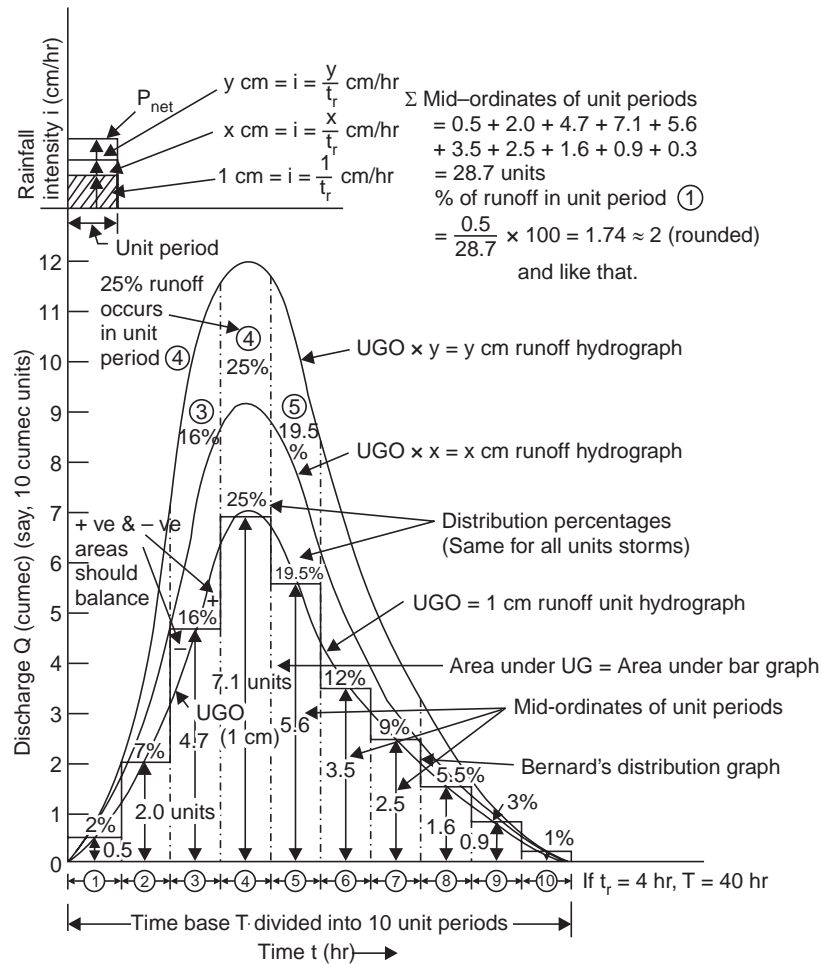


Fig. 5.13 Distribution percentages same for all unit storms

Limitation of the Unit Hydrograph. Certain limitations are inherent in the unit hydrograph theory. The runoff hydrograph reflects the combined effects of rainfall factors, loss factors and physiographic factors. The design storm continuing for several unit periods may not have the same areal distribution for each time increment. Storm movements also affect the proportions of the unit hydrograph if the basin is large. Hence, the unit hydrograph can not be applied for basins larger than 5000 km². For basins larger than 5000 km², unit hydrographs for the principal sub-areas or sub-basins are developed and the hydrographs of runoff determined for each sub-area. These hydrographs are then combined, through flood routing procedure, to get the resulting hydrograph at the required section.

Derivation of Unit Hydrograph

Example 5.1 (a) The runoff data at a stream gauging station for a flood are given below. The drainage area is 40 km². The duration of rainfall is 3 hours. Derive the 3-hour unit hydrograph for the basin and plot the same.

<i>Date</i>	<i>Time (hr)</i>	<i>Discharge (cumec)</i>	<i>Remarks</i>
1-3-1970	2	50	
	5	47	
	8	75	
	11	120	
	14	225	
	17	290	← Peak
	20	270	
	23	145	
2-3-1970	2	110	
	5	90	
	8	80	
	11	70	
	14	60	
	17	55	
	20	51	
	23	50	

State the peak of the unit hydrograph you derive.

Solution See Table 5.1.

5.5 UNIT HYDROGRAPH FROM COMPLEX STORMS

Unit hydrographs from complex storms, involving varying intensities of rain can be obtained by considering the complex storm as successive unit storms of different intensities and the runoff hydrograph (due to complex storm) as the result of superposition of the successive storm hydrographs. The ordinates of each storm hydrograph are obtained as ‘the storm intensity times the corresponding ordinate of the unit hydrograph’ as shown in Fig. 5.14. The unit hydrograph ordinates u_1, u_2, \dots are thus obtained by writing a series of equations for each of the ordinates Q_1, Q_2, \dots of the runoff hydrograph (due to complex storm) and successively solving them. In Fig. 5.14,

$$Q_1 = xu_1 \quad \therefore \quad u_1 = ?$$

$$Q_2 = xu_2 + yu_1 \quad \therefore \quad u_2 = ?$$

$$Q_3 = xu_3 + yu_2 + zu_1 \quad \therefore \quad u_3 = ?$$

and so on. Thus, the t_r -hour unit graph ordinates can be determined. Although the method is straight forward, errors will creep in due to the assumptions on the intensity and duration of rainfall and deduction of an assumed base flow; many trials are required to get a reasonable unit graph.

Table 5.1 Derivation of the 3-hour unit hydrograph. Example 5.1

<i>Date</i>	<i>Time (hr)</i>	<i>TRO</i> ¹ (cumec)	<i>BFO</i> ² (cumec)	<i>DRO</i> ² (3)–(4) (cumec)	<i>UGO</i> ⁴ (5) ÷ <i>P</i> _{net} (cumec)	<i>Time</i> ⁵ from begin- ning of surface run off (hr)
1	2	3	4	5	6	7
1-3-1970	2	50	50	—	—	—
	5	47	47	0	0	0
	8	75	46	29	1.09	3
	11	120	45	75	2.82	6
	14	225	45	180	6.77	9
	17	290	45	245	9.23	12
	20	270	46	224	8.44	15
2-3-1970	23	145	48	97	3.65	18
	2	110	50	60	2.26	21
	5	90	53	37	1.39	24
	8	80	54	26	0.98	27
	11	70	57	13	0.49	30
	14	60	60	0	0	33
	17	55	55	—	—	—
	20	51	51	—	—	—
23	50	50	—	—	—	
				Σ <i>DRO</i> = 986 cumec		

¹*TRO*—Total runoff ordinate = gauged discharge of stream

²*BFO*—Base flow ordiante read from graph separation line shown in Fig. 5.10 (a). $N = 0.83 A^{0.2} = 0.89(40)^{0.2} = 1.73$ days = $1.73 \times 24 = 41.4$ hr from peak, which is seen not applicable here; hence an arbitrary separation line is sketched.

³*DRO*—Direct runoff ordinate = *TRO*—*B.F.O.*

⁴*UGO*—Unit hydrograph ordinate

$$= \frac{DRO}{P_{net}}; P_{net} = \frac{\Sigma DRO \cdot t}{A} = \frac{986(3 \times 60 \times 60)m^3}{40 \times 10^6 m^2}$$

$$= 0.266 m = 26.6 cm$$

⁵Time from beginning of direct surface runoff is at 5 hr on 1-3-1970, which is reckoned 0 hr for unit hydrograph. The time base for unit hydrograph is 33 hours.

The 3-hour unit hydrogryph is plotted in Fig. 5.10 (b) to a different vertical scale and its peak is 9.23 cumec.

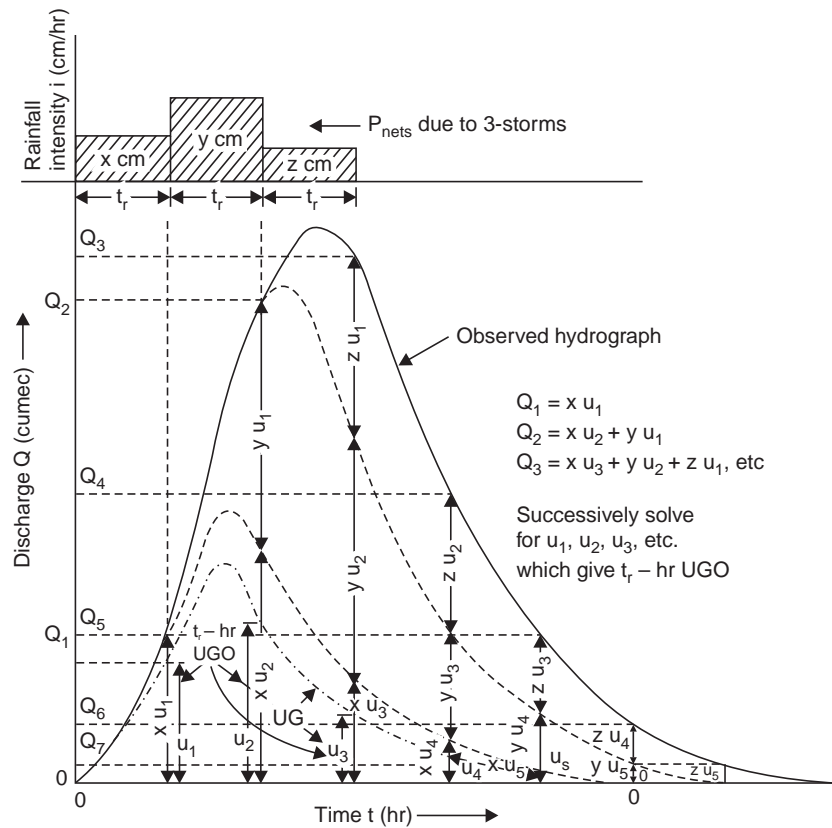


Fig. 5.14 Derivation of unit hydrograph from multi-period storms

Example 5.2 The stream flows due to three successive storms of 2.9, 4.9 and 3.9 cm of 6 hours duration each on a basin are given below. The area of the basin is 118.8 km². Assuming a constant base flow of 20 cumec, derive a 6-hour unit hydrograph for the basin. An average storm loss of 0.15 cm/hr can be assumed.

Time (hr):	0	3	6	9	12	15	18	21	24	27	30	33
Flow (cumec):	20	50	92	140	199	202	204	144	84.5	45.5	29	20

Solution Let the 6-hour unit hydrograph ordinates be $u_0, u_1, u_2, u_3, u_4, \dots, u_7$ at 0, 3, 6, 12, ..., 21 hours, respectively. The direct runoff ordinates due to the three successive storms (of 6 hours duration each) are obtained by deducting the base of flow of 20 cumec from the streamflows at the corresponding time intervals as shown in Table 5.2. The net storm rains are obtained by deducting the average storm loss as

$$\begin{aligned}
 0-6 \text{ hr: } x &= 2.9 - 0.15 \times 6 = 2 \text{ cm} \\
 6-12 \text{ hr: } y &= 4.9 - 0.15 \times 6 = 4 \text{ cm} \\
 12-18 \text{ hr: } z &= 3.9 - 0.15 \times 6 = 3 \text{ cm}
 \end{aligned}$$

The equations can be easily arrived by entering in a tabular column and successively solving them. The 6-hr unit hydrograph ordinates are obtained in the last column; of course the ordinates are at 3-hr intervals since the streamflows are recorded at 3-hr intervals. The

Table 5.2 Derivation of a 6-hour unit hydrograph from a complex storm, Example 5.2.

Time (hr)	DRO due to**			Equation		Solution 6-hr UGO
	1st storm UGO × x	2nd storm UGO × y	3rd storm UGO × z	Total DRO = TRO — BFO		
0	$u_0 = 0$	—	—	$0 = 20 - 20$		$u_0 = 0$
3	u_1	—	—	$2u_1 = 50 - 20$		$u_1 = 15$
6	u_2	0	—	$2u_2 = 92 - 20$		$u_2 = 36$
9	u_3	$4u_1$	—	$2u_3 + 4u_1 = 140 - 20$		$u_3 = 30$
12	u_4	$4u_2$	0	$2u_4 + 4u_2 + 0 = 199 - 20$		$u_4 = 17.5$
15	u_5	$4u_3$	$3u_1$	$2u_5 + 4u_3 + 3u_1 = 202 - 20$		$u_5 = 8.5$
18	u_6	$4u_4$	$3u_2$	$2u_6 + 4u_4 + 2u_2 = 204 - 20$		$u_6 = 3$
21	u_7	$4u_5$	$3u_3$	$2u_7 + 4u_5 + 3u_3 = 144 - 20$		$u_7 = 0$
24		$4u_6$	$3u_4$	$4u_6 + 3u_4 = 84.5 - 20$		$\Sigma u = 110$
27		$4u_7$	$3u_5$	$4u_7 + 3u_5 = 45.5 - 20$		check for
30			$3u_6$	$3u_6 = 29 - 20$		UGO derived
33			$3u_7$	$3u_7 = 20 - 20$		above

**Except the first column, all other columns are in cumec.

**x = 2 cm, y = 4 cm, z = 3 cm

last four equations in the table serve to check some of the *UGO*'s derived. Another check for the *UGO*'s derived is that the area under the *UG* should give a runoff volume equivalent to 1 cm, *i.e.*,

$$\frac{\Sigma u t}{A} = 1 \text{ cm, in consistent units}$$

$$\Sigma u = \text{sum of the } UGO\text{'s} = 110 \text{ cumec}$$

$$\therefore \frac{110 (3 \times 60 \times 60)}{118.8 \times 10^6} = 0.01 \text{ m, or } \mathbf{1 \text{ cm}}$$

Hence, the *UGO*'s derived are correct and is plotted in Fig. 5.15 (b).

The *UGO*'s can also be derived by the method of least squares (Snyder, 1955) by writing the direct runoff ordinate (*Q*) as

$$Q = a + b_1x_1 + b_2x_2 + b_3x_3 + \dots + b_ix_i \quad \dots(5.5)$$

where $a = 0$ theoretically, $b_i = u_i$, and x_1, x_2, x_3, \dots are the rainfall excesses (P_{net}) in successive periods. In the least square technique, data from a number of flood events, for which values of Q_i and x_i are established, are used to develop a set of average values of $b_i (= u_i)$ and the unit hydrograph derived. The method is more elegant but laborious.

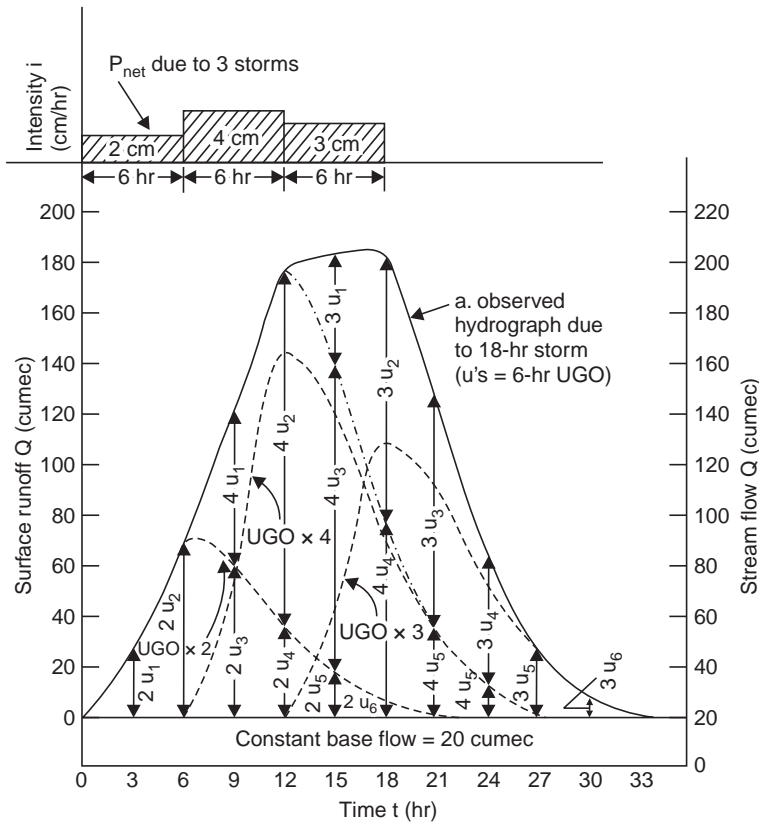


Fig. 5.15 (a) Derivation of 6-hr UG from 18-hr complex storm hydrograph (Example 5.2)

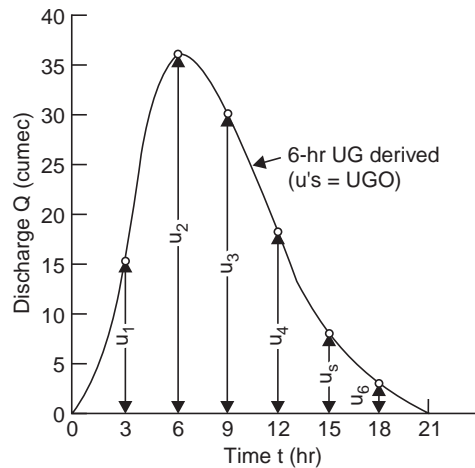


Fig. 5.15 (b) 6-hr unit hydrograph (derived) (Example 5.2)

Matrix method for unit hydrograph derivation from complex storm

In matrix notation

$$PU = Q$$

where $P = \begin{bmatrix} p_1 & 0 & 0 & 0 & 0 & 0 \\ p_2 & p_1 & 0 & 0 & 0 & 0 \\ p_3 & p_2 & p_1 & 0 & 0 & 0 \\ 0 & p_3 & p_2 & p_1 & 0 & 0 \\ 0 & 0 & p_3 & p_2 & 0 & 0 \\ 0 & 0 & 0 & p_3 & 0 & 0 \end{bmatrix}$ $U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ 0 \\ 0 \end{bmatrix}$ $Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix}$

By deducting base flow from the stream flow hydrograph Q is known, and with the known rainfall excess P , the unit hydrograph U can be computed as

$$U = P^{-1} Q$$

The inverse for the precipitation matrix (P^{-1}) exists only if it is a square matrix with a non-vanishing determinant. However, by the use of the transpose of the precipitation matrix, (P^T), a square matrix is obtained as

$$P^T P U = P^T Q$$

and the unit hydrograph matrix is obtained as

$$U = (P^T P)^{-1} P^T Q$$

Usually, the solution of the unit hydrograph matrix is performed on a digital computer (Newton, 1967). Most computer installations have packaged programmes to work with large systems and for determining the inverse and transpose of matrices.

Also, the number of streamflow hydrograph ordinates (n) is given by

$$n = j + i - 1$$

where j = number of UGO

i = number of periods of rainfall

In Example 5.2, $n = 12$ ordinates at 3-hr interval (DRO of the first and last being zero); $i = 6$ periods of 3-hr interval ($= 3 \times 2$), then

$$n = j + i - 1; \quad 12 = j + 6 - 1$$

$$\therefore \quad j = 7 \text{ UGO}$$

Thus, the unit hydrograph consists of 7 ordinates at 3-hr intervals (of course, $u_0 = 0$, and $u_7 = 0$).

As an assignment, the student is advised first to become well versed with the matrix algebra and then apply the matrix method to derive the 6-hr UGO at 3-hr interval from the streamflow data given in Example 5.2.

Average Unit Hydrograph. It is better if several unit graphs are derived for different isolated (single peaked) uniform intensity storms. If the durations of storms are different, the unit hydrographs may be altered to the same duration (sect or art). From several unit hydrographs for the same duration, so obtained, an average unit hydrograph can be sketched by computing the average of the peak flows and times to peak and sketching a median line, (Fig. 5.16), so that the area under the graph is equal to a runoff volume of 1 cm.

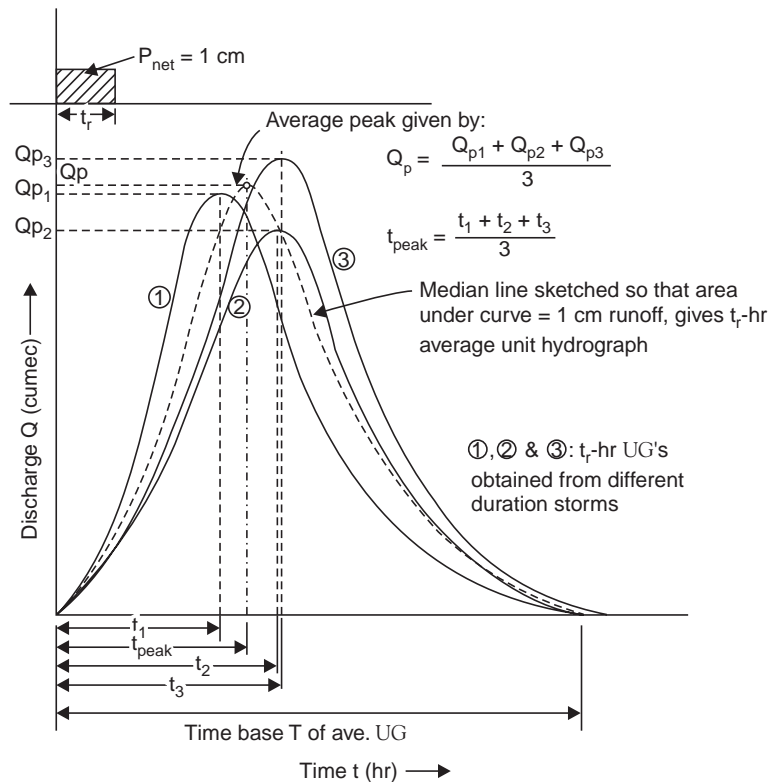


Fig. 5.16 Average unit hydrograph

Alteration of Unit Hydrograph Duration. It becomes necessary, in computation of flood hydrographs, that the duration of the unit graph available should be altered to suit the duration of the design storm (to be used for obtaining the flood hydrograph). Two cases arise:

Case (i) *Changing a Short Duration Unit Hydrograph to Longer Duration.* If the desired long duration of the unit graph is an even multiple of the short, say a 3-hour unit graph is given and a 6-hour unit graph is required. Assume two consecutive unit storms, producing a net rain of 1 cm each. Draw the two unit hydrographs, the second unit graph being lagged by 3 hours. Draw now the combined hydrograph by superposition. This combined hydrograph will now produce 2 cm in 6 hours. To obtain the 6-hour unit graph divide the ordinates of the combined hydrograph by 2, Fig. 5.17. It can be observed that this 6-hour unit graph derived has a longer time base by 3 hours than the 3-hour unit graph, because of a lower intensity storm for a longer time.

Example 5.3 *The following are the ordinates of a 3-hour unit hydrograph. Derive the ordinates of a 6-hour unit hydrograph and plot the same.*

<i>Time (hr)</i>	<i>3-hr UGO (cumec)</i>	<i>Time (hr)</i>	<i>3-hr UGO (cumec)</i>
0	0	15	9.4
3	1.5	18	4.6
6	4.5	21	2.3
9	8.6	24	0.8
12	12.0		

Solution

Table 5.3 Derivation of the 6-hour unit hydrograph. (Example 5.3)

<i>Time (hr)</i>	<i>3-hr UGO (cumec)</i>	<i>3-hr UGO (logged) (cumec)</i>	<i>Total (2) + (3) (cumec)</i>	<i>6-hr UGO (4) ÷ 2 (cumec)</i>
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
0	0		0	0
3	1.5	0	1.5	0.7
6	4.5	1.5	6.0	3.0
9	8.6	4.5	13.1	6.5
12	12.0	8.6	20.6	10.3
15	9.4	12.0	21.4	10.7
18	4.6	9.4	14.0	7.0
21	2.3	4.6	6.9	3.4
24	0.8	2.3	3.1	1.5
27		0.8	0.8	0.4

The 6-hour unit graph derived as above is shown in Fig. 5.17.

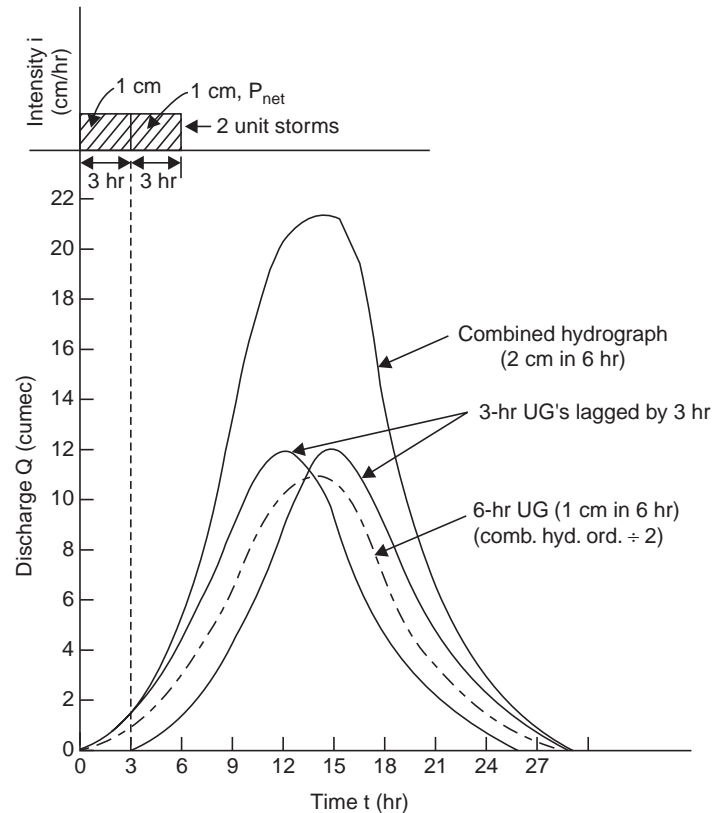


Fig. 5.17 Changing a short duration UG to a long multiple duration

Case (ii) *Changing a Long Duration Unit Hydrograph to a Shorter Duration by S-curve Technique*

5.6 S-CURVE METHOD

S-curve or the summation curve is the hydrograph of direct surface discharge that would result from a continuous succession of unit storms producing 1 cm in t_r -hr (Fig. 5.18). If the time base of the unit hydrograph is T hr, it reaches constant outflow (Q_e) at T hr, since 1 cm of net rain on the catchment is being supplied and removed every t_r hour and only T/t_r unit graphs are necessary to produce an S-curve and develop constant outflow given by,

$$Q_e = \frac{2.78 A}{t_r} \quad \dots(5.6)$$

where Q_e = constant outflow (cumec)

t_r = duration of the unit graph (hr)

A = area of the basin (km^2)

Given a t_r -hour unit graph, to derive a t_r' -hour unit graph ($t_r' \geq t_r$)—Shift the S-curve by the required duration t_r' along the time axis. The graphical difference between the ordinates of the two S-curves, i.e., the shaded area in Fig. 5.18 represents the runoff due to t_r' hours rain at